A New Hip at the Sea-Side - Medical Tourism and Hospital Competition

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Preliminary

Abstract

This paper studies the impact of patient mobility on quality in a hospital market with a regulated price in a two-country-extension of the framework of Brekke et al. (2011a). Countries may differ in treatment price and/or hospital density. Under identical treatment prices patient mobility decreases quality in the home (sending) country and increases quality in the foreign (receiving) country. Under identical densities of hospitals patient mobility increases (decreases) quality in the home (sending) country, if the fraction of mobile patients is sufficiently low (high) and/or the number of hospitals is sufficiently high (low), and increases quality in the foreign (receiving) country.

JEL Classification: H42, I11, I18, L13

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1 Introduction

This paper studies the impact of patient mobility on quality in a hospital market with a regulated price in a two-country-extension of the framework of Brekke et al. (2011).

In the European Union, health policy, including the general design of health care systems, falls in the member states’ competence (Treaty on the Functioning of the European Union (TFEU), Art. 168). As a result, health care systems in Europe differ e.g. in the mode of financing, the number of sickness funds, the degree of regulation and the coverage of services. At the same time, freedom of movement requires that

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people working, traveling or retiring abroad can also receive medical service in another
country (Rosenmoeller et al., 2006). In addition, these differences between health care
systems can make traveling abroad specifically for treatment attractive, if prices are
lower, quality is higher, waiting lists are shorter or other treatments than at home are
available.

Directive 2011/24/EU provides citizens in European Union with the right to choose
among health care providers across all EU member states. Countries have to reimburse
patients for cross-border medical treatment when this treatment is also covered in the
patients’ home country.

One of the first papers to study the effect of patient mobility is Brekke et al. (2011b).
Studying patient mobility in a Hotelling model with two regions differing in technology,
they find that the effects of patient mobility depends on the transfer payment: Under a
transfer payment below marginal cost patient mobility leads to a race-to-the-bottom in
quality and lower welfare in both regions, under a transfer payment equal to marginal
cost quality and welfare remain unchanged in the high-skill region, but the low-skill
region benefits.

Against this background this paper studies the impact of patient mobility on quality
in a hospital market with a regulated price in a two-country-extension of the framework
of Brekke et al. (2011a). Countries may differ in treatment price and/or hospital density.
Under identical treatment prices patient mobility decreases quality in the home (sending)
country and increases quality in the foreign (receiving) country. Under identical densities
of hospitals patient mobility increases (decreases) quality in the home (sending) country,
if the fraction of mobile patients is sufficiently low (high) and/or the number of hospitals
is sufficiently high (low), and increases quality in the foreign (receiving) country.

The remaining of the paper is organized as follows: In the next section, the two-
country model of a hospital market with a regulated price is presented and the equilibria
with and without patient mobility are derived. Section 3 studies the impact of patient
mobility on quality. Section 4 concludes.

2 The Model

Using the two-country-extension of the framework of Brekke et al. (2011a), consider the
market for an elective hospital treatment in two countries. The two countries, home and
foreign \((j = H, F)\), are represented by two Salop circles. The outer circle represents the
home country, where \(n_H\) hospitals are located equidistantly on a circle with a circumference
of 1. The inner circle represents the foreign country, where \(n_F\) hospitals are located
equidistantly on a circle with a circumference of $B$, $0 < B < 1$, see Figure 1.

Figure 1: Home and Foreign

In both countries, there is a unit mass of patients uniformly distributed on the circle. A patient demands one treatment from the 'most preferred' hospital or no treatment at all.

A patient located at $z_k$, $k = H, F$, obtains the following utility from a treatment at hospital $i$ located at $x_{j(i)}$ in country $j$

$$U = \begin{cases} 
    v + s_{j(i)} - p_j - t |z_k - x_{j(i)}| & \text{if } k = j \\
    v + s_{j(i)} - p_j - t |z_k - x_{j(i)}| - f & \text{if } k \neq j,
\end{cases} \tag{1}$$

where $v$ is gross utility from treatment, $s_{j(i)}$ is the quality level at hospital $i$ in country $j$, $p_j$ is the regulated price in country $j$, and $t$ is traveling cost. Crossing the border and travelling to a hospital in the other country entails fixed cost $f$. Patients pay the price $p_j$ per treatment privately.

As in Brekke et al. (2011b), the objective function for hospital $i$ in country $j$ is given as

$$\phi_{j(i)} = T_j + p_j q_{j(i)} - c(s_{j(i)}) + \alpha B_{j(i)}, \tag{2}$$

where $T_j$ is a lump-sum transfer, $q_{j(i)}$ is the demand for treatments at hospital $i$, $c(s_{j(i)}) = \frac{1}{2} s_{j(i)}^2$ is the cost of providing quality $s_{j(i)}$, $B_{j(i)}$ is the benefit to patients treated at
hospital \( i \) and \( \alpha \), \( 0 < \alpha < 1 \) is the degree of altruism of the hospital. Assume that \( t > 2\alpha \).

### 2.1 Equilibrium without Patient Mobility

First consider the case without patient mobility, where patients only seek treatment in the country of residence.

In country \( H \), the patient indifferent between treatment at hospital \( i \) and hospital \( i + 1 \) is given by

\[
v + s_{H(i)} - p_H - tz_H = v + s_{H(i+1)} - p_H - t \left( \frac{1}{n_H} - z_H \right),
\]

which yields

\[
z_H = \frac{s_{H(i)} - s_{H(i+1)}}{2} + \frac{t}{n_H}.
\]

Total demand for hospital \( i \) is given as \( q_{H(i)} = 2z_H \).

The objective function for hospital \( i \) is given as

\[
\phi_{H(i)} = T_H + p_H (2z_H) - \frac{1}{2} s_{H(i)}^2 + \alpha 2 \int_0^z (v + s_{H(i)} - p_H - tz) \, dz.
\]

Maximizing (5) with respect to quality and applying symmetry yields the equilibrium treatment quality:

\[
s_H = \frac{\alpha (t + 2vn_H) + 2n_Hp_H (1 - \alpha)}{2n_H (t - \alpha)}.
\]

A higher price \( p_H \) or a higher patient valuation \( v \) increase treatment quality. In addition, a higher degree of altruism \( \alpha \) increases treatment quality. A higher degree of competition, as measured by a higher number of hospitals \( n_H \) or lower traveling cost \( t \), decreases treatment quality. Brekke et al. (2011a) also show that lower traveling cost \( t \) reduces quality if the competitive segment is sufficiently large.

In country \( F \), the patient indifferent between treatment at hospital \( i \) and hospital \( i + 1 \) is given by

\[
v + s_{F(i)} - p_F - tz_F = v + s_{F(i+1)} - p_F - t \left( \frac{B}{n_F} - z_F \right),
\]

which yields

\[
z_F = \frac{s_{F(i)} - s_{F(i+1)} + B \frac{t}{n_F}}{2t}.
\]
Total demand for hospital \( i \) is given as \( q_F(i) = 2z_F \).

The objective function for hospital \( i \) is given as

\[
\phi_F(i) = T_F + p_F (2z_F) - \frac{1}{2} s_F(i) + \alpha 2 \int_0^{x_F} (v + s_F(i) - p_F - tz) \, dz.
\]  

Maximizing (9) with respect to quality and applying symmetry yields the equilibrium quality:

\[
s_F = \frac{\alpha (Bt + 2vn_F) + 2n_Fp_F (1 - \alpha)}{2n_F (t - \alpha)}.
\]

(10)

Treatment quality increases in country size \( B \). As in country \( H \), a higher price \( p_F \) or a higher patient valuation \( v \) increase treatment quality. In addition, a higher degree of altruism \( \alpha \) increases treatment quality. A higher degree of competition, as measured by a higher number of hospitals \( n_F \) or lower traveling cost \( t \), decreases treatment quality.

The quality difference between treatments in \( H \) and \( F \) is given as

\[
\Delta = s_F - s_H = \frac{\alpha(Bn_H - n_F) + 2n_Fn_H (1 - \alpha)(p_F - p_H)}{2n_Fn_H (t - \alpha)}.
\]

(11)

For identical prices, treatment quality in country \( F \) is higher than in country \( H \), if the density of hospitals is higher \((\frac{B}{n_F} > \frac{1}{n_H})\), thus competition among hospitals is stronger in country \( F \). This is, the quality-reducing effect of more hospitals \((\frac{\partial s}{\partial n} < 0)\) is lower in country \( F \). For countries of identical size with the same number (and density) of hospitals, treatment quality is higher in the country with the higher price.

An increase in country size \( B \) of country \( F \) increases \( s_F \), while leaving \( s_H \) unchanged. For \( s_F > s_H \), an increase in \( B \) increases the quality difference; for \( s_F < s_H \), an increase in \( B \) decreases the quality difference. An increase in the price difference \( p_F - p_H \) increases the quality difference, as a higher price involves higher quality. An increase in the number of hospitals in country \( H \) decreases \( s_H \), while leaving \( s_F \) unchanged. For \( s_F > s_H \), an increase in \( n_H \) increases the quality difference; for \( s_F < s_H \), an increase in \( n_H \) decreases the quality difference. The effect of an increase in \( n_F \) on the quality difference is vice versa. An increase in traveling cost \( t \) decreases both quality levels, with the effect on the higher level being stronger: For \( s_F > s_H \), an increase in \( t \) decreases \( s_F \) by more than \( s_H \) and thus also the quality difference; for \( s_F < s_H \), in \( t \) decreases \( s_H \) by more than \( s_F \) and the quality difference decreases. Vice versa, an increase in the degree of altruism \( \alpha \) increases the quality difference if \( s_F > s_H \), and increases the quality difference if \( s_H > s_F \).
2.2 Equilibrium with Patient Mobility

Now consider the case with patient mobility, where some patients consider treatment abroad. An asterisk is used to denote variables associated with patient mobility.

Assume that a fraction \( \mu \) of patients in the home country is mobile and may seek treatment in country \( F \), if quality is higher and/or the price is lower. As in Brekke et al. (2011b), for this fraction of patients the fixed cost of travelling abroad is normalized to zero. For the remaining fraction \( (1 - \mu) \) the cost of travelling to the other country is prohibitively high. Assume also hospitals in the two countries are located in such a way, that is from all locations in the home country the distance to (and hence the cost of travelling to) a hospital in the foreign country is the same. See Figure 2 for an illustration of the patient decision. The patient located at \( x \) will weight the benefits and costs of treatment at hospital \( H(i) \) against the benefits and costs of treatment at hospital \( F(i) \). Travelling from country \( H \) to country \( F \) is equivalent to changing from the outer Salop circle to the inner Salop circle.

\[
v + s^*_H(i) - p^*_H - t\zeta^*_H = v + s^*_F(i) - p^*_F - t \left( \frac{1}{2n_H} - \zeta^*_H \right), \tag{12}
\]

Figure 2: Patient Mobility

Assume that patients first travel along the arc and then cross the border.

In country \( H \), the mobile patient indifferent between treatment at hospital \( i \) in country \( H \) and hospital \( i \) in country \( F \), \( \zeta^*_H \), is given by
which yields
\[
\zeta_H^* = \frac{s_{H(i)}^* - s_{F(i)}^* + p_{F}^* - p_{H}^* + \frac{t}{2n_H}}{2t}.
\] (13)

Total demand from the mobile fraction for hospital \(i\) given as \(q_{H(i)} = 2\zeta_H^*\). The location of indifferent patient from the immobile fraction, \(z_H^*\), in country \(H\) is the same as under no mobility.

In country \(H\), the objective function for hospital \(i\) is given as
\[
\phi_{H(i)} = T_H + p_{H}^* \left((1 - \mu) 2z_H^* \mu + \mu^2 \zeta_H^* - \frac{1}{2} s_{H(i)}^* \right.
+ (1 - \mu) \alpha \int_0^{z_H^*} \left(v + s_{H(i)}^* - p_{H}^* - tz\right) dz
+ \mu \alpha \int_0^{z_H^*} \left(v + s_{H(i)}^* - p_{H}^* - tz\right) dz.
\] (14)

Maximizing (14) with respect to quality and applying symmetry yields the best response function:
\[
s_H^* = \frac{\alpha \left(t + 2vn_H\right) + 2n_H p_{H}^* (1 - \alpha) - \alpha \mu \left(\frac{t}{2} + s_{H}^* n_H + n_H \left(p_{H}^* - p_{F}^*\right)\right)}{2n_H \left(t - \alpha \left(1 + \frac{1}{2} \mu\right)\right)}.
\] (15)

The best response function increases in both prices and decreases in the quality level in country \(F\), if the fraction of mobile patients is sufficiently low.

In country \(F\), the objective function for hospital \(i\) is given as
\[
\phi_{F(i)} = T_F + p_{F}^* \left(z_F^* \mu + \mu^2 \left(\frac{t}{2} - \zeta_H^*\right)\right) - \frac{1}{2} s_{F(i)}^* \right)
+ \alpha \int_0^{z_F^*} \left(v + s_{F(i)}^* - p_{F}^* - tz\right) dz
\]
\[\]
\[+ \mu \alpha \int_0^{z_H^*} \left(v + s_{F(i)}^* - p_{F}^* - tz\right) dz.
\] (16)

Maximizing (16) with respect to quality and applying symmetry yields the best response function:
\[
s_F^* = \frac{\alpha \left(Bt + 2vn_F (1 + \mu)\right) + 2n_F p_{F}^* ((1 + \mu) - \alpha) + \alpha \mu \left(\frac{1}{2} t n_F - n_F s_{H}^* + n_F \left(p_{H}^* - 3p_{F}^*\right)\right)}{2n_F \left(t - \alpha \left(1 + \frac{3}{2} \mu\right)\right)}.
\] (17)

The best response function increases in both prices and decreases in the quality level in
country $H$, if the fraction of mobile patients is sufficiently low. Note that quality levels are strategic substitutes, if the fraction of mobile patients is sufficiently low. Otherwise they are strategic complements.

Equilibrium quality levels are given as

$$s^*_H = \frac{\alpha (t + 2vn_H) + 2n_HP_H (1 - \alpha) - \alpha \mu \frac{3 + t - \alpha(4 + \mu)}{2(t - \alpha)}}{2n_H (t - \alpha) - \alpha \mu \left(2 - \frac{\alpha \mu}{2(t - \alpha)}\right)},$$

$$s^*_F = \frac{\alpha (Bt + 2vn_F) + 2n_FP_F (1 - \alpha) - \mu (-2t + 3\alpha + 3\alpha - 4\alpha^2 + \alpha \mu - \alpha^2 \mu)}{2n_F (t - \alpha) - \alpha \mu \left(2 - \frac{\alpha \mu}{2(t - \alpha)}\right)}.$$

(18)

3 Quality and Patient Mobility

This section investigates the consequences of patient mobility for quality provision in both countries.

Proposition 1 summarizes the effect of the change from no mobility to patient mobility:

**Proposition 1** Suppose that treatment prices are identical in both countries. Then i) quality in the home country is lower under patient mobility, ii) quality in the foreign country is higher under patient mobility. Suppose the densities of hospitals are identical in both countries. Then i) quality in the home country is higher (lower) under patient mobility, if the fraction of mobile patients is sufficiently low (high) and/or the number of hospitals is sufficiently high (low) ii) quality in the foreign country is higher under patient mobility.

**Proof.** See Appendix. ■

If treatment prices are identical in both countries, patient mobility is entirely driven by quality differences. This makes providing higher quality more attractive. However, in the home country, patient mobility also includes a negative effect on quality. This effect
dominates and quality in the home country is lower under patient mobility. Quality in the foreign country is higher under patient mobility.

If both countries have identical densities of hospitals, the negative effect on quality in the home is decreased by a smaller fraction of mobile patients and a high number of hospitals. If the fraction of mobile patients is sufficiently low and/or the number of hospitals is sufficiently high (low) quality in the home country is higher under patient mobility. Quality in the foreign country is higher under patient mobility.

Proposition 2 summarizes the effect of an increase in the mobile fraction in the population:

**Proposition 2** Suppose that treatment prices are identical in both countries. Then i) quality in the home country decreases in the mobile fraction, ii) quality in the foreign country increases in the mobile fraction. Suppose the densities of hospitals are identical in both countries. Then i) quality in the home country decreases in the mobile fraction, ii) quality in the foreign country increases in the mobile fraction.

**Proof.** See Appendix.

### 4 Conclusion

In this paper, I have studied the impact of patient mobility on quality in a hospital market with a regulated price in a two-country-framework. Countries may differ in treatment price and/or hospital density. Under identical treatment prices patient mobility decreases quality in the home (sending) country and increases quality in the foreign (receiving) country. Under identical densities of hospitals patient mobility increases (decreases) quality in the home (sending) country, if the fraction of mobile patients is sufficiently low (high) and/or the number of hospitals is sufficiently high (low), and increases quality in the foreign (receiving) country.

In a next step, the impact of price regulation on quality under patient mobility needs to be assessed. On the one hand, as higher prices make providing higher quality more attractive, the higher prices can maybe reduce the negative impact of mobility on quality provision in the home (sending) country. On the other, a higher prices makes seeking treatment abroad for patients more attractive. Correspondingly, the total effect of changes in prices needs to be studied.
References


5 Appendix

5.1 Equilibrium without Patient Mobility

\[ s_H = \frac{a(t+2vn_H)+2n_Hp_H(1-\alpha)}{2n_H(t-\alpha)} \]

\[ \frac{\partial s_H}{\partial H} = \frac{1-\alpha}{t-\alpha} > 0 \]

\[ \frac{\partial s_H}{\partial \alpha} = \frac{a t}{2n_H(t-\alpha)} < 0 \]

\[ \frac{\partial s_H}{\partial v} = \frac{a^2 + 2a\alpha + 2n_H p_H(1-\alpha)}{2n_H(t-\alpha)} > 0 \]

\[ \Delta = s_F - s_H = \frac{a(Bn_H - n_F) + 2n_H p_H(1-\alpha)(p_F - p_H)}{2n_F n_H(t-\alpha)} \]

Assume \( p_F = p_H = p \)

Then \( \Delta = s_F - s_H = \frac{a(Bn_H - n_F)}{2n_F n_H(t-\alpha)} > 0 \) if \( \frac{B}{n_F} > \frac{1}{n_H} \)

Assume \( \frac{1}{n_H} = \frac{B}{n_F} \)

Then \( \Delta = 2n_F n_H(1-\alpha)(p_F - p_H) > 0 \) if \( p_F > p_H \)

\[ \frac{\partial \Delta}{\partial B} = \frac{a t}{2n_F(t-\alpha)} > 0 \]

\[ \frac{\partial \Delta}{\partial p_F} = \frac{1-\alpha}{t-\alpha} > 0 \]

\[ \frac{\partial \Delta}{\partial n_H} = \frac{a t}{2n_Fn_H(t-\alpha)} > 0 \]

\[ \frac{\partial \Delta}{\partial n_F} = -\frac{B t}{2n_F^2(t-\alpha)} < 0 \]

\[ \frac{\partial \Delta}{\partial \alpha} = \frac{a^2 (Bn_H - n_F) + 2n_H p_H(1-\alpha)(p_F - p_H)}{2n_F n_H(t-\alpha)^2} < 0 \]

\[ \Delta = \frac{\partial \Delta}{\partial \alpha} > 0 \]

\[ \frac{\partial \Delta}{\partial \alpha} = \frac{\partial \Delta}{\partial \alpha} > 0 \]

\[ \frac{\partial \Delta}{\partial t} = \frac{\partial \Delta}{\partial t} > 0 \]

5.2 Equilibrium with Patient Mobility

\[ s_H^* (s_F^*) = \frac{a(t+2vn_H)+2n_H p_H^*(1-\alpha)-\alpha p_H^* (1+\frac{2}{1+\mu}) n_H (p_F^* - p_H^*)}{2n_H(t-\alpha(1+\frac{1}{1+\mu}))} \]

\[ \frac{\partial (s_H^* (s_F^*))}{\partial p_H^*} = \frac{1-\alpha(1+\frac{1}{1+\mu})}{(t-\alpha(1+\frac{1}{1+\mu}))} > 0 \] if \( \mu < \frac{2(1-\alpha)}{\alpha} \) and \( \mu < \frac{2(t-\alpha)}{\alpha} \)
\[
\frac{\partial s_H^* (s_H^*)}{\partial p_F} = \alpha \frac{\mu}{2(t-\alpha(1+\frac{\mu}{2}))} > 0 \text{ if } \mu < \frac{2(1-\alpha)}{\alpha}
\]
\[
\frac{\partial s_H^* (s_H^*)}{\partial p_H} = -\alpha \frac{\mu}{2(t-\alpha(1+\frac{\mu}{2}))} < 0 \text{ if } \mu < \frac{2(1-\alpha)}{\alpha}
\]
\[
s_F^* (s_H^*) = \frac{\alpha B + 2p_{\beta F}(1+\mu) + 2n_F p_F^* (1+\mu) - \alpha \mu \left( \frac{1}{2} \frac{n_F}{n_F^*} - n_F s_H^* + n_F (p_H^* - 3p_F^*) \right)}{2n_F (t-\alpha(1+\frac{\mu}{2}))}
\]
\[
\frac{\partial s_H^* (s_H^*)}{\partial p_F} = \alpha \frac{\mu}{2(t-\alpha(1+\frac{\mu}{2}))} > 0 \text{ if } \mu < \frac{2(1-\alpha)}{3\alpha}
\]
\[
\frac{\partial s_H^* (s_H^*)}{\partial p_H} = \frac{(1+\mu-\alpha(1+\frac{\mu}{2}))}{(t-\alpha(1+\frac{\mu}{2}))} > 0 \text{ if } \mu < \left( \frac{1}{2} - \frac{1}{\alpha-1} \right) \land \mu < \frac{2(1-\alpha)}{3\alpha}
\]
\[
\frac{\partial s_H^* (s_H^*)}{\partial p_H} = -\alpha \frac{\mu}{2(t-\alpha(1+\frac{\mu}{2}))} < 0 \text{ if } \mu < \left( \frac{1}{2} - \frac{1}{\alpha-1} \right)
\]

Assume \( p_F = p_H = p \).

Then \( s_H - s_H^* = \alpha \mu \left( \frac{t(2\alpha - 2) + \alpha \mu (t - 2\alpha) + 2n_H (p_F - p_H)}{2n_F (t-\alpha(1+\frac{\mu}{2}))} \right) > 0 \),

\[
\frac{2n_H (p_F - p_H)^* (t-\alpha)(1-\mu)}{2n_F (t-\alpha(1+\frac{\mu}{2}))} < 0
\]

Assume \( \frac{1}{n_H} = \frac{p}{n_F} \Leftrightarrow n_F = Bn_H \).

Then \( s_H - s_H^* = \alpha \mu \left( \frac{t(2\alpha - 2) + \alpha \mu (t - 2\alpha) + 2n_H (p_F - p_H)}{2n_F (t-\alpha(1+\frac{\mu}{2}))} \right) > 0 \),

\[
t(t-\alpha)^2 - \alpha \mu (t-\alpha) - 2n_H (p_F - p_H) (-t + \alpha - t\alpha + \alpha \mu + t^2) - 2\mu n_H (\alpha p_H - p_F) + 2t v H \mu H < 0
\]

if \( \mu \leq \frac{2n_H (p_F - p_H) (t-\alpha)^2}{2n_F (t-\alpha(1+\frac{\mu}{2}))} \),

\[
t(t-\alpha)^2 - \alpha \mu (t-\alpha) - 2n_H (p_F - p_H) (-t + \alpha - t\alpha + \alpha \mu + t^2) - 2\mu n_H (\alpha p_H - p_F) + 2t v H \mu H < 0
\]

if \( n_H \geq \frac{2n_H (p_F - p_H) (t-\alpha)^2}{2n_F (t-\alpha(1+\frac{\mu}{2}))} \).

Assume \( p_F = p_H = p \).

\[
\frac{\partial s_H^*}{\partial \mu} = -\alpha n_H (t-\alpha) \left( \frac{2(t-\alpha)^2 - \alpha \mu (4-t\alpha - 3\alpha \mu)}{2n_F n_H (t-\alpha)^2 - 4\alpha \mu (4-t\alpha - 3\alpha \mu)} \right)
\]

\[
\frac{\partial s_H^*}{\partial \mu} = \frac{1}{2} \left( \frac{2n_F n_H (2-t\alpha)^2 - 4\alpha \mu (2-t\alpha)^2 - \alpha \mu^2}{n_F n_H (2-t\alpha)^2 - 4\alpha \mu (2-t\alpha)^2 - \alpha \mu^2} \right) > 0
\]

Assume \( \frac{1}{n_H} = \frac{1}{n_F} \Leftrightarrow n_F = Bn_H \).

\[
\frac{\partial s_H^*}{\partial \mu} = -\frac{1}{2} \left( \frac{(2(t-\alpha)^2 - \alpha \mu (4-t\alpha - 3\alpha \mu) + 8t v H \mu H (t-\alpha(1+\mu))}{n_H (2(t-\alpha)^2 - 4\alpha \mu (2-t\alpha)^2 - \alpha \mu^2)} \right)
\]

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\[ + \frac{1}{2} \Omega \frac{2n_H \left( \left( 2(t-1)(t-\alpha)^2 - \alpha^2 \mu(3\mu+4) \right) (\mu_H - \mu_H) - \mu H (4(t-2\alpha) - \alpha \mu(4-\alpha)) + \alpha \mu \mu_H (4(t-\alpha) - (3\mu+4)) \right)}{n_H (2(t-\alpha)^2 - 4\alpha \mu (t-\alpha) + \alpha^2 \mu^2)^2} < 0 \]

\[ \frac{\partial s^*_F}{\partial \mu} = \frac{2n_H (2(t-\alpha)^2 - \alpha^2 \mu (4(t-\alpha) + \mu (t-3\alpha))) + 4 \tan \mu H (2(t-\alpha)^2 - \alpha \mu (2(t-\alpha) - \alpha \mu))}{2n_H (2(t-\alpha)^2 - 4\alpha \mu (t-\alpha) + \alpha^2 \mu^2)^2} \]

\[ + \frac{2n_H \alpha \mu_H (1) (2(t-\alpha)^2 - \alpha^2 \mu^2) + \mu H (2(t-\alpha - 3t\alpha) (t-\alpha)^2 + \alpha \mu H (4(t-1) (t-\alpha) - \alpha \mu(2t+\alpha + ta)))}{2n_H (2(t-\alpha)^2 - 4\alpha \mu (t-\alpha) + \alpha^2 \mu^2)^2} > 0. \]