Merger control on two-sided markets: is there need for an efficiency defense?*

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April 25, 2015

Abstract

We study horizontal mergers on two-sided markets between horizontally differentiated platforms. We provide a theoretical analysis of the merger’s price effect based on the amount of cost savings it generates, the behavior of outsider platforms, and the size of cross-group network effects. We point out differences as compared with the standard, one-sided merger analysis, and also discuss the merger control policy implications.

Keywords: horizontal merger, two-sided markets, cost savings, merger control

JEL codes: L41, D82, K21

*The authors gratefully acknowledge the financial support of this research by the NET Institute, http://www.NETinst.org. The paper benefited from comments received at the ASSET 2014 conference in Aix-en-Provence, the 2014 Law&Economic Policy Workshop on Antitrust for Platform and Network Markets in Nanterre, and the MaCCI 2015 conference in Mannheim. The usual disclaimer applies.

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1 Introduction

This is a theoretical paper discussing the price effects of horizontal mergers on two-sided markets. In a four-platform framework allowing for horizontal differentiation, we analyze the final price effect for customers taking into account both the cost savings from the merger and the pricing behavior of outsider platforms. In contrast to standard, one-sided merger theory, the impact of cost savings on prices is not monotonic, and depends on the cross-network externalities. In particular, the market’s ‘two-sidedness’ makes the efficiency defense redundant if the merger takes place between distant, less substitutable platforms. However, in the case of merger between neighbor, closely substitutable platforms, the efficiency defense is still necessary if the cross-group externality is weak. Our results are useful for competition authorities facing the challenge of assessing mergers between competing platforms on two-sided markets.

Starting with Rochet and Tirole (2002, 2003), two-sided markets have been the object of increasing research focus over the past fifteen years, in particular due to the fact that these industries often exhibit business conducts and outcomes that would be sub-optimal on traditional (one-sided) markets. The much quoted example that springs to mind is that of firms, even monopolies, setting prices below cost on one side of the platform so as to maximize overall profit thanks to increasing demand on the other side. In other words, the indirect (or cross-group) network effects at work on two-sided markets possibly lead to outcomes not usually predicted by standard analysis. This holds not only from a positive stand, but also from a normative one, to the extent that the previously-mentioned behavior may simply not represent predatory pricing, and as such should not be subject to antitrust vetoing. And while much has been accomplished regarding unilateral pricing strategies, much is left to be done for the study of coordinated behavior such as pricing following a horizontal merger.

Mergers between rivals typically give rise to enhanced market power and higher prices, thus harming customers. But on two-sided markets another effect may reverse this outcome: the cross-group externalities granting users increased utility from having access to a greater pool of business partners on the other side of the platform may neutralize, and even outweigh the utility loss due to the price increase. As a result, the merger could actually be welfare enhancing rather than welfare detrimental (Evans, 2003). Moreover, it is even possible for post-merger prices to be lower than before merger, due to the fact that the merging platforms internalize the effect of a price increase on the merger partner platform - in other words, the same indirect externality may reverse the typical post-merger incentive to increase prices to exploit market power (see Chandra and Collard-Wexler (2009) and Leonello (2010)). Following this type of argument,
Evans and Schmalensee (2007) remind that traditional merger analysis may still apply when the degree of "two-sidedness" (or the size of network externalities, equivalently) is low enough, whereas Evans and Noel (2008) point out some crucial difficulties raised by using conventional methods to analyze mergers in two-sided markets\(^1\).

Understanding and correctly predicting the outcome of mergers on two-sided markets is increasingly relevant from the practical, public policy viewpoint (Economides 2008, 2010), given the recent surge in such cases for competition authorities\(^2\). Interestingly enough, the two-sided nature of the market does not systematically play a role in the decision\(^3\), and when it does, it plays against the parties: the Norwegian media merger between Edda Media and A-Pressen was cleared in 2012 conditional on structural remedies/asset divestitures on certain local/geographic markets, whereas the Deutsche Börse-NYSE Euronext deal was banned in 2012 due to insufficient cost savings to compensate for the supposedly likely post-merger price increase. Such decisions clearly go against the conclusions of most of the literature dealing with the topic, which is incidentally empirical, and generally agrees that, at least for the cases studied, horizontal mergers on two-sided market do not lower welfare\(^4\). Clearly, there is need to further study mergers on two-sided markets, all the more so that few papers address this from the theoretical point of view.

Chandra and Collard-Wexler (2009) specifically study the post-merger pricing in two-sided markets based on a theoretical model: a modified Hotelling duopoly where consumers are assumed to be single-homing, whereas advertisers may advertise in several newspapers. The key finding is that increased concentration may not lead to higher prices on either side, because the resulting monopoly may actually choose to set lower prices. This result is however conditioned on pricing below marginal cost on the reader side: if newspapers sell their content at a price below marginal cost, then additional readers are only valuable to the extent that the revenues which could be made by selling their attention to advertisers are greater than the subsidy. The

\(^1\)The Lerner index for instance does not hold, and merger price-simulation methods that are now commonly used for traditional markets turn out to be misspecified when applied to multi-sided markets. See Affeldt and al. (2013) for how to adapt the UPP test to two-sided markets.

\(^2\)See for the instance the merger between the Dutch yellow pages directories (Case No. 6246/European Directories, 2008), the Edda Media and A-Pressen merger in Noway (Konkurranse Tilsynet Case 2011/0925 MAB BMBE) or the Deutsche Borse-NYSE Euronext deal (EC Case No. M.6166-2012).

\(^3\)Incidentally, although a new edition of the US Merger Guidelines was released in 2010, replacing the previous Guidelines from 1992, there was no mention of two-sided markets.

fact that the data on the Canadian newspaper industry corroborates the absence of price increase following horizontal mergers does not however rule out alternate explanations: it is perfectly possible that some mergers were accompanied by efficiency gains/cost savings, which by virtue of lower costs for the merged parties allowed prices to remain unchanged.

A second theoretical attempt to address optimal pricing after a horizontal mergers on two-sided markets is Leonello (2010). In a merger-to-monopoly scenario, the monopolist will offer advertisers in one newspaper the opportunity to advertise also in the other one as part of the merger deal. For a single price, the advertiser can now reach twice as many consumers as before, which is referred to as 'interoperability'. Leonello (2010) shows that the introduction of advertising bundling by the monopolist increases the incentives to keep prices low on at least one side of the market, because the interoperability increases the margin which the newspaper can charge on advertising, and it thereby becomes profitable to reduce prices on the consumer side in order to stimulate demand. Overall, welfare could increase following a merger, and this result is obtained absent efficiency gains.

We depart from these papers by focusing in contrast on the role played by the merger cost savings for the post-merger pricing strategy of platforms, and thereby ultimately for the consumer welfare impact of the merger. This is arguably relevant for competition authorities, since they are bound to explicitly balance the efficiency gains from a merger against its anti-competitive effect in order to decide whether to ban or to clear it. Our setting also allows to compute the equilibrium prices of outsider platforms, and we therefore provide the overall optimal post-merger pricing schedule as a function of the amount of indirect, cross-group externalities. Arguably, we do not perform a full-fledged welfare analysis of the merger since we focus on the price behavior of both merging and non-merging platforms. Nonetheless, to the extent that we also look into the merger’s profitability, both internal and external, we do provide some insight into the likely evolution of welfare as a result of the merger.

For this, we use the spatial circular framework with exogenous and symmetrical differentiation à la Salop. We assume perfect symmetry between the two sides of the market, and

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5It could also be that some mergers were driven by motives other than increased market power, such as for instance empire-building or political motives (Anderson and McLaren, 2012).


7Formally, the paper closest to ours is Brito (2005), except for the two-sided perspective of course. Gal-Or and Dukes (2006) also use the Salop setting to address horizontal media mergers, but they focus exclusively on the role of bargaining power on one side of the market, i.e. between advertisers and radio stations in particular, for the profitability of non-consolidating deals.
compute and compare optimal prices set by both insider and outsider platforms in two different cases: bilateral merger between adjacent or distant platforms. We show that two cases arise for the merger between adjacent platforms. With relatively low cross-group externality, post-merger prices fall iff the merger generates enough cost savings. Moreover, the bilateral merger is always profitable for the insiders, but harms outsiders whenever the post-merger price falls. The opposite case, of high cross-group externality, yields different results: post-merger prices as well as the profits of outsider platforms fall whatever the amount of merger cost savings. However, the merger is internally profitable iff the efficiency gains are high enough. On the other hand, the bilateral merger between distant platforms improves consumer welfare and harms outsider platforms regardless of the merge cost savings, but is internally profitable only for high enough efficiency gains. Finally, in terms of policy implications, we conclude that the efficiency defense may be required for mergers between adjacent platforms, but is not necessary for mergers involving distant platforms.

Before going on to our model, let us note that we perform a significantly more general analysis than what has been previously done in the literature. To start with, we allow for outsiders and no longer consider merger to monopoly. This enables a more thorough consumer welfare analysis since the final price effect depends on the outsiders’ pricing strategy, and also makes possible to consider different types of merger: between neighbor, or, on the contrary, distant platforms. Finally, we allow for cost savings, which enables the study of the relationship between merger efficiency gains and indirect network effects from the point of view of optimal post-merger pricing.

The rest of the paper is organized as follows: first we present the framework, then we begin the merger analysis by computing the price equilibrium before merger. We then consider the two types of mergers, between adjacent and distant platforms respectively, and compute the post-merger equilibria. We discuss the policy implications before concluding. All proofs are grouped in the Appendix at the end of the paper.

We provide upon request the Maple computations for the equilibrium prices and profits - they are very space consuming and as such are left out the current version.
2 The model

2.1 The framework

Consider a four-platform market. The platforms, denoted \( k \in \{A, B, C, D\} \), compete in prices and are equidistantly located on the unit circular market. We assume exogenous locations: \( A \) is located in \( 0(1) \), \( B \) in \( 1/4 \), \( C \) in \( 1/2 \) and \( D \) in \( 3/4 \). Each platform produces or sells the same type of product/service to two groups of customers, enabling them to interact. Let there be \( N_i \) customers uniformly distributed along each side \( i \in \{1, 2\} \) of the market. We assume single-homing throughout the paper: each customer has access to the other side of the market through one platform only. Denote \( v \) the reservation price, the same for all buyers. They all have inelastic demand, buying only one unit of the good/service iff the price is less than the reservation price. The platforms do not spatially discriminate their buyers, and so they use mill pricing. Each customer will choose the platform offering the lowest total price, equal to the sum of the transportation cost and the sale price. We assume linear transportation cost \( t_i d \) on each side \( i \), where \( t_i \) is the constant unit cost and \( d \) the distance from a platform to the customer’s location\(^9\).

The net utility for a customer on side \( i \) using platform \( k \) writes as follows:

\[
U^k_i = v + a_i n^k_j - p^k_i - t_i d_{i,k}
\]

where \( a_i n^k_j \) is the indirect network externality (a customer patronizing platform \( k \) on side \( j \) induces consumer utility equal to \( a_i \) on side \( i \)), and \( d_{i,k} \) is the distance between the customer on side \( i \) and the platform \( k \). For instance,

\[
U^A_1 = v + a_1 N_2 x^A_2 - p^A_1 - t_1 x^A_1
\]

where \( x^A_1 \) stands for the distance on the side 1 of the market traveled by the customer located in \( x^A_1 \) to join the platform \( A \). Let \( a_i \geq 0, N_i \geq 0 \), and assume the intrinsic utility \( v \) to be high enough to always guarantee full market coverage.

\(^9\)Arguably, the choice of linear transport cost may seem restrictive, to the extent that the initial spatial pattern that we consider on each side of the market, that of equidistant locations, has only been rationalized as a location equilibrium for the pure-strategy location-then-price game in the case of quadratic transport cost (Economides 1989). However, we do not deal with location choices, and neither do we allow for relocation, and therefore maintain our choice of a linear transport cost for obvious tractability reasons.
2.2 Pre-merger analysis

Before merger all platforms operate with the same constant unit production cost, $c$. Each platform $k$ will maximize total profit it makes on both sides of the market by optimally setting prices $p^k_t$. In order to determine the pre-merger price equilibrium, one needs first to establish the system of demands, and for this one has to write the demand served by platform $k$ on each side.

The marginal customer $x_{i}^{A(D)}$ located between $A$ and $D$ on side $i$ is defined by

$$v + a_{i}N_jx^{A}_{j} - p^{A}_{i} - t_{i}x^{A(D)}_{i} = v + a_{i}N_jx^{D}_{j} - p^{D}_{i} - t_{i}\left(\frac{1}{4} - x^{A(D)}_{i}\right),$$

while the marginal customer between $A$ and $B$ on the same side of the market is defined by

$$v + a_{i}N_jx^{A}_{j} - p^{A}_{i} - t_{i}x^{A(B)}_{i} = v + a_{j}N_jx^{B}_{j} - p^{B}_{i} - t_{i}\left(\frac{1}{4} - x^{A(B)}_{i}\right).$$

The total demand for platform $A$ on side $i$ is therefore given by $x^{A(D)}_{i} + x^{A(B)}_{i}$, where

$$x^{A(D)}_{i} = \frac{4a_{i}N_jx^{A}_{j} - 4p^{A}_{i} - 4a_{i}N_jx^{D}_{j} + 4p^{D}_{i} + t_{i}}{16t_{i}}$$

and

$$x^{A(B)}_{i} = \frac{4a_{i}N_jx^{A}_{j} - 4p^{A}_{i} - 4a_{i}N_jx^{B}_{j} + 4p^{B}_{i} + t_{i}}{16t_{i}}.$$  

Solving the system of total demands yields the individual demands for each platform $k$ for each side of the market $i$ as functions of all prices. Total profits write therefore as follows:

$$\Pi^{A} = (p^{A}_{i} - c)N_1\left(x^{A(D)}_{1} + x^{A(B)}_{1}\right) + (p^{A}_{2} - c)N_2\left(x^{A(D)}_{2} + x^{A(B)}_{2}\right),$$

$$\Pi^{B} = (p^{B}_{i} - c)N_1\left(x^{B(C)}_{1} + x^{B(A)}_{1}\right) + (p^{B}_{2} - c)N_2\left(x^{B(C)}_{2} + x^{B(A)}_{2}\right),$$

$$\Pi^{C} = (p^{C}_{i} - c)N_1\left(x^{C(D)}_{1} + x^{C(B)}_{1}\right) + (p^{C}_{2} - c)N_2\left(x^{C(D)}_{2} + x^{C(B)}_{2}\right),$$

$$\Pi^{D} = (p^{D}_{i} - c)N_1\left(x^{D(A)}_{1} + x^{D(C)}_{1}\right) + (p^{D}_{2} - c)N_2\left(x^{D(A)}_{2} + x^{D(C)}_{2}\right).$$

and the vector of equilibrium prices $((p^{A}_{1})^*, (p^{B}_{1})^*, (p^{C}_{1})^*, (p^{D}_{1})^*, (p^{A}_{2})^*, (p^{B}_{2})^*, (p^{C}_{2})^*, (p^{D}_{2})^*)$ solves the system of FOCs, i.e.

$$(p^{k}_{i})^* = \arg\max_{p^k_{i}} \left(N_ix^k_{i}(p^k_{i} - c) + N_jx^k_{j}(p^k_{j} - c)\right), i, j = 1, 2, k = A, B, C, D.$$
Due to the symmetry between the four platforms on each side of the market, one has at equilibrium that \((p^*_1) = (p^*_B) = (p^*_C) = (p^*_D) \equiv p^*_1\), and \((p^*_2) = (p^*_B) = (p^*_C) \equiv p^*_2\), where:

\[
p^*_1 = \frac{2 (t_1)^2 t_2 - 18c a_2 N_1 a_1 N_2 - 3N_2 a_2 t_1 t_2 + 8c t_1 t_2 - 5a_2 N_1 a_1 N_2 t_1 + 6 (N_2)^2 (a_2)^2 N_1 a_1}{2(4t_1 t_2 - 9a_2 a_1 N_1 N_2)},
\]

\[
p^*_2 = \frac{2 (t_2)^2 t_1 + 8c t_1 t_2 - 3N_1 a_1 t_1 t_2 - 5a_2 N_1 a_1 N_2 t_2 + 6 (N_1)^2 (a_1)^2 N_2 a_2 - 18ca_2 N_2 a_1 N_1}{2(4t_1 t_2 - 9a_2 a_1 N_1 N_2)}.
\]

Equilibrium profits are in turn given by:

\[
\Pi^*_i = \frac{(2N_1 (t_1)^2 t_2 - 3N_1 N_2 a_2 t_1 t_2 - 5a_2 (N_1)^2 a_1 a_2 N_2 t_1 + 6 (N_2)^2 (a_2)^2 (N_1)^2 a_1 + 2N_2 t_1 (t_2)^2)}{8(4t_1 t_2 - 9a_2 a_1 N_1 N_2)} - \frac{-3N_2 N_1 a_1 t_1 t_2 - 5a_2 N_1 a_1 (N_2)^2 t_2 + 6 (N_1)^2 (a_1)^2 a_2 (N_2)^2 t_2 + 6 (N_1)^2 (a_1)^2 a_2 (N_2)^2}{8(4t_1 t_2 - 9a_2 N_1 a_1 N_2)}.
\]

### 3 Merger analysis

The model we propose will remain relatively simple and above all tractable thanks to several assumptions and simplifications.

We leave aside the merger’s impact on locations, so we only consider exogenous differentiation. We equally neglect the possibility for the merger to trigger changes for the number of active outlets, or changes in the users’ willingness to pay - although the merger does give access to more users/business partners on the other side of the market. We also neglect any vertical differentiation issues (see Fan 2013 and Jeziorski 2014 for examples of empirical analyses taking this into account). In other words, we simply assume that the merger will change the ownership pattern in the industry, and lead to a joint pricing decisions for the merging platforms, but will not alter the type of equilibrium that the platforms play, nor change the users’ preferences.

Finally, in order to keep the analysis analytically tractable, in what follows we use several simplifying assumptions. First, we only consider perfectly symmetric differentiation between the four platforms, i.e. let \(t_1 = t_2 = t\). Secondly, we assume equal market size for the two sides connected by the platforms: \(N_1 = N_2 = N\). Lastly, we take the cross-group externalities to be identical for the two sides of the market: \(a_1 = a_2 = a\), meaning that customers on each side of the market value the other side participation with the same intensity. Denote then \(Na = z\).

Given all the above simplifications, pre-merger prices become

\[
p^*_1 = p^*_2 = p^* = \frac{t^2 - 3tz - 6zc + 4tc + 2z^2}{2(2t - 3z)},
\]
whereas all pre-merger profits now equal

$$\Pi^* = \frac{N (t - 2z)(t - z)}{4(2t - 3z)}.$$  

The Hessian matrix writes

$$H = \frac{N}{(t^2 - z^2)(t^2 - 4t^2)} \begin{bmatrix} t(5z^2 - 2t^2) & 3z(2z^2 - t^2) \\ 3z(2z^2 - t^2) & t(5z^2 - 2t^2) \end{bmatrix},$$

leading to $z < 0.5t$ or $0.666 < z < t$ for the SOCs to be satisfied.

One more remark is necessary before turning to the merger analysis. As mentioned in the introduction, our four-platform framework enables the analysis of two types of merger: between adjacent and distant platforms respectively. In both cases we compare post-merger prices and profits with those before. The purpose is to make clear to what extent the ‘two-sidedness’ of the market changes results as compared with the ‘standard’ merger analysis performed on one-sided markets.

The main insights of the latter can be summarized as follows: without cost savings, a merger between neighbor firms competing in prices on the circular market increases all prices, but the farther away the outsiders, the lower their price increase (Levy and Reitzes, 1992). Moreover, such a merger is profitable for the merging firms, although the outsiders generally also gain from it, but possibly to different extents, depending on their distance/relative position from the insiders (Brito, 2003). In turn, the merger between distant firms should not affect prices, nor profits, in the absence of cost savings, simply because for the market-power price-increasing effect to arise, the two merger firms need some captive demand, which they cannot enjoy unless they are neighbors. Thus, the spatial literature on horizontal mergers between firms competing in prices on the circular market (Levy and Reitzes (1992), Brito (2003, 2005)) concludes that a bilateral merger between neighbors necessarily involves a unilateral market power effect and as such is always profitable, whereas a merger between non adjacent firms can only be motivated either by cost savings or possibly to better sustain collusion. Let us now also briefly recall the usual post-merger trade-off arising whenever a merger may lead to cost savings. On the one hand, the market power effect pushes the price upwards, so as to increase profits over the residual captive demand served by the two merging partners. But on the other hand, the cost savings increase their productive efficiency relative to their rivals, enabling the insiders to lower their prices, attract more customers, and thereby increase their profits. The net price effect for the insider firms naturally depends on the amount of cost savings, and in what follows we shall focus on identifying the amount of cost savings that make the merger neutral from the point of view of customers.
3.1 Merger between adjacent platforms

Let platforms $A$ and $B$ merge. As before mentioned, we do not allow for platform relocation, nor changes in the customers’ willingness to pay. The merger simply allows customers on one side of the market to reach more customers on the other side, although they still connect via a single platform. The merger does not involve shutting down any of the platforms, so post-merger there will still be eight prices to be determined: the joint pricing decision for $A$ and $B$ will lead to different values for $p_1^A, p_1^B, p_2^A$ and $p_2^B$, triggering in response different values for $p_1^C, p_1^D, p_2^C$ and $p_2^D$. In fact, the market turns into an asymmetric triopoly, since we allow for cost savings/efficiency gains from merger: the merged platform will operate with a lower constant unit cost as compared with the remaining outsider platforms. Denote $c - \Delta$ the unit cost for the group $A + B$, where $\Delta \in [0, c]$. The main purpose of our analysis will be to determine the impact of the amount of cost savings $\Delta$ on the change in prices due to the merger.

The merged entity will now maximize

$$\Pi^A + \Pi^B = (p_1^A - c + \Delta)N \left(x_1^{A(D)} + x_1^{A(B)}\right) + (p_2^A - c + \Delta)N \left(x_2^{A(D)} + x_2^{A(B)}\right)$$
$$+ (p_1^B - c + \Delta)N \left(x_1^{B(C)} + x_1^{B(A)}\right) + (p_2^B - c + \Delta)N \left(x_2^{B(C)} + x_2^{B(A)}\right),$$

whereas the platforms $C$ and $D$ go on maximizing their stand-alone profits:

$$\Pi^C = (p_1^C - c)N \left(x_1^{C(D)} + x_1^{C(B)}\right) + (p_2^C - c)N \left(x_2^{C(D)} + x_2^{C(B)}\right),$$
$$\Pi^D = (p_1^D - c)N \left(x_1^{D(A)} + x_1^{D(C)}\right) + (p_2^D - c)N \left(x_2^{D(A)} + x_2^{D(C)}\right).$$

Due to the symmetry between the insider platforms, as well as between the outsiders, we obtain the following optimal post-merger prices for the insider and outsider platforms respectively:

$$\tilde{p}^{A+B} = \frac{10ct - 16zc - 11zt - 6t\Delta + 10z\Delta + 4t^2 + 7z^2}{2(5t - 8z)},$$
$$\text{and } \tilde{p}^C = \frac{10ct - 16zc - 9zt - 2t\Delta + 4z\Delta + 3t^2 + 6z^2}{2(5t - 8z)}.$$

Note that: $\tilde{p}^{A+B} - c + \Delta \geq 0, \forall \Delta \in [0, c]$, but $\tilde{p}^C - c \geq 0$ iff $\Delta \leq \frac{3}{2}(t - z) \equiv \overline{\Delta}$.

Let $f(\Delta, z, c)$ stand for the post-merger/pre-merger price difference for the insider platforms on either side of the market:

$$f(\Delta, z, c) = \tilde{p}^{A+B} - p^* = \frac{3t - 5z}{(4t - 6z)(5t - 8z)} \left(6z\Delta - 4t\Delta - 2zt + t^2 + z^2\right).$$
By the same token, the outsider-platform price difference between merger and no-merger on either side of the market writes

\[ h(\Delta, z, c) = \tilde{p}^C - p^* = \frac{1}{2} \left( \frac{t - 2z}{2(t - 3z)(5t - 8z)} \right) \left( 6z\Delta - 4t\Delta - 2zt + t^2 + z^2 \right). \]

Based on the monotonicity of \( f \) and \( h \) (see details in Appendix 1), and restricting the analysis to \( z \in [0, 0.5t) \cup (0.75t, t) \) satisfying the SOCs, the following results hold:

**Lemma 1** There exists a threshold of cost savings \( \tilde{\Delta} \) such that the merger leaves all prices unchanged.

It is straightforward to check that \( \tilde{\Delta} = \frac{(z-t)^2}{2(t-3z)} \) solves for \( f(\Delta, z, c) = h(\Delta, z, c) = 0 \).

**Proposition 1** Consider a bilateral merger between adjacent platforms on a perfectly symmetric four-firm two-sided market; then

(i) for a weak indirect externality, i.e. \( z \in [0, 0.5t) \), the merger leads to lower prices iff the merger generates enough cost savings (\( \Delta \in [\Delta, \tilde{\Delta}] \));

(ii) for a strong indirect externality, i.e. \( z \in (0.75t, t) \), the merger always leads to lower prices regardless of the amount of merger efficiency gains.

Proposition 1 details the merger’s impact on consumers’ welfare depending on the intensity of the cross-group externality between the two sides of the market. If this externality is weak, post-merger prices fall for high enough cost savings generated by the merger. Instead, the merger’s efficiency gains no longer matter when the externality is strong, because in that case post-merger price decrease anyway. In other words, Proposition 1 points out a certain degree of substitutability between the merger’s efficiency gains and the market’s two-sidedness: the higher the latter, the lower the importance of cost savings.

To better grasp this result, recall that the cross-group externality favors customers to the extent that it provides incentives for firms to lower their prices so as to better benefit from the increase in demand that it gives rise to - basically, it works similarly to cost savings from the point of view of firms’ profits. As a result, a weak indirect externality cannot fully compensate the market power effect of the merger, and it takes high enough cost savings for the insiders to lower their price. However, as soon as the indirect externality is strong enough, the merger cost savings are less needed for the price to diminish. The final price effect for customers is

\[ 10 \text{The same type of result holds for quantity competition - see Bond (1996) for a Cournot model showing that competitors are not affected by the merger when the equilibrium price does not change.} \]
unambiguous, thanks to the strategic complementarity between the pricing decisions of insider and outsider firms: the latter will also lower their prices. Note finally that it is the intensity of the externality that is essential for the pricing behavior of the merging firms, or, equivalently, the results in Proposition 1 are indeed driven by the market’s two-sidedness. For this, consider the case of zero cost savings: on one-sided markets, post-merger prices necessarily increase, whereas here customers still benefit from the merger provided that the externality is high enough.

The implications for competition policy are straightforward, and on this point our analysis formally confirms the intuition of Evans and Schmalensee (2007) that the ‘traditional’ merger analysis still applies when the degree of ‘two-sidedness’ is low enough. According to Proposition 1, the efficiency defense is necessary only if the indirect externality is low, because in that case the proof that the efficiency gains are high enough would allow the merger to be cleared (since then customers would not be harmed). However, the possible necessity of an efficiency defense does not imply its opportunity as well. For this, one has to look into the merger profitability. Indeed, consider the case where customers benefit from the merger, but the latter is not profitable for the merging firms: then the merger would likely not be submitted in the first place, and as a result there would be no occasion for the efficiency defense to apply. Below we discuss the merger’s impact on the firms’ profits, so as to draw proper conclusions for the applicability of the efficiency defense.

Thanks to our simplifying assumption of perfect symmetry between the two sides of the market, the profits for the insiders and outsiders now equal

\[
\Pi^A + B = \frac{1}{2} N \left( \frac{4t\Delta - 11zt - 6z\Delta + 4t^2 + 7z^2}{(t - z)(5t - 8z)^2} \right)
\]

and

\[
\Pi^C = \Pi^D = \frac{1}{4} N \left( \frac{(2t - 3z)(3z - 3t + 2\Delta)^2}{(5t - 8z)^2} \right) \equiv \Pi^O
\]

Therefore, the profit differentials for insiders and outsiders respectively write as follows:

\[
\tilde{\Pi}^{A+B} - 2\Pi^* = N \left( \frac{4(2t - 3z)^3 \Delta^2 + 4(4t - 7z)(t - z)(2t - 3z)^2 \Delta + (19z^2 - 23zt + 7t^2)(t - z)^3}{2(t - z)(8z - 5t)^2(2t - 3z)} \right)
\]

and

\[
\tilde{\Pi}^O - \Pi^* = N \left( \frac{(4t - 6z)^2 \Delta^2 - 12(t - z)(3z - 2t)^2 \Delta + (t - z)^3(11t - 17z)}{4(t - z)(2t - 3z)^2(8z - 5t)^2} \right).
\]

Thus the following holds:

**Proposition 2** Consider a bilateral merger between adjacent platforms on a perfectly symmetric four-firm two-sided market; then
(i) for a weak indirect externality, i.e. \( z \in [0, 0.5t) \), the merger is always profitable for the insiders but it harms outsiders iff the merger generates enough cost savings to lower prices, i.e. \( \Delta \in [\hat{\Delta}, \Delta] \);

(ii) for a strong indirect externality, i.e. \( z \in (0.75t, t) \), the merger is profitable for the insiders iff it generates enough cost savings \( (\Delta \in [\Delta^*, \Delta]) \), but it always harms outsiders.

Proposition 2 details the merger’s impact on the profits of all firms in the industry as a function of the intensity of the indirect externality. Two cases arise. First, when the cross-group externality is weak, the merger between neighbor platforms is always internally profitable, but it harms outsiders whenever it benefits customers. This situation basically extends its corresponding one-sided counterpart, because a merger between neighbor firms competing in prices is always profitable for them (see Levy and Reitzes (1992) or Brito (2005)), all the more so when it involves cost savings. Note moreover that this means that the market power effect of the merger (i.e. exploiting its captive demand) relatively dominates, since the merger is profitable even with zero indirect externality and zero cost savings. However, outsider platforms are now harmed by the merger if it generates enough cost savings. Secondly, when the cross-group externality is strong, it takes high enough efficiency gains for the merger to be profitable. However, the previous outcome for outsiders still holds, they lose profits. This is actually easily explained by the merger’s impact on prices. To begin with, whenever the merging platforms lower their prices, outsider firms will do the same, due to the strategic complementarity of pricing decisions, but less than the insiders, since they do not enjoy cost savings. As a result, they will lose some demand in the process, and therefore the combined final effect for their profits is unambiguous: they are harmed by the merger whenever customers benefit from it through lower prices.\(^{11}\)

At this point several observations are worth making. First, the indirect externality and the cost savings appear somewhat complementary from the point of view of the merger’s internal profitability: a weak indirect externality and low cost savings do not prevent the merger from taking place, but the insiders need high enough efficiency gains to merge despite a strong indirect externality. This second case basically reminds of a Cournot-like merger, where the market power effect is not enough to guarantee profitability due to business stealing\(^{12}\), thus making cost

\(^{11}\)Neven and Röller (2005) were among the first to stress the implications for merger control of the fact that consumers and outsiders have always divergent interests, both with price and quantity competition.

\(^{12}\)The business stealing effect stands for the fact that the insiders’ price drop does not bring about enough demand to increase profits, because the outsider platform respond by also lowering their price.
savings a *sine qua non* condition for the merger to take place. However, here we only deal with price-setting platforms, and moreover, the business stealing effect is completely reversed to favor customers and hurt outsiders: the latter are always harmed by the merger, and do not benefit from it even though the cost savings may be too low for the merger to be profitable. Arguably, this is due to the presence of a strong indirect externality.

Given Propositions 1 and 2, the outcome in terms of applicability of the efficiency defense is straightforward. Whenever the cross-group externality between the two sides of the market is weak, bilateral mergers between neighbor platforms are always profitable, and as such will be submitted. However, they only improve consumers’ welfare for high enough cost savings, thus making the efficiency defence necessary within a consistent merger control. In turn, a strong indirect externality guarantees that the customers benefit from the merger, but the latter only takes place for high enough cost savings: the efficiency defense is thus redundant. Below we summarize these merger control implications:

**Corollary 1** Consider a bilateral merger between adjacent platforms with symmetrical cross-group externalities; then the efficiency defense

(i) should be used for weak indirect externality, since the submitted mergers only benefit customers for high enough cost savings;

(ii) is useless for strong indirect externality, since all submitted merger benefits customers.

### 3.2 Merger between distant platforms

Let us now consider the alternative merger, between platforms $A$ and $C$. Again, we only allow for cost savings/efficiency gains from merger: the merged platform will operate with a lower constant unit cost as compared with the remaining outsider platforms. Denote now $c - \Lambda$ the unit cost for $A + C$, where $\Lambda \in [0, c]$.

The merged entity will now maximize

$$
\Pi^A + \Pi^C = (p_1^A - c + \Lambda)N \left( x_1^{A(D)} + x_1^{A(B)} \right) + (p_2^A - c + \Lambda)N \left( x_2^{A(D)} + x_2^{A(B)} \right) + (p_1^C - c + \Lambda)N \left( x_1^{C(D)} + x_1^{C(B)} \right) + (p_2^C - c + \Lambda)N \left( x_2^{C(D)} + x_2^{C(B)} \right),
$$

whereas the platforms $B$ and $D$ go on maximizing their stand-alone profits:

$$
\Pi^B = (p_1^B - c)N \left( x_1^{B(C)} + x_1^{B(A)} \right) + (p_2^B - c)N \left( x_2^{B(C)} + x_2^{B(A)} \right),
$$

$$
\Pi^D = (p_1^D - c)N \left( x_1^{D(A)} + x_1^{D(C)} \right) + (p_2^D - c)N \left( x_2^{D(A)} + x_2^{D(C)} \right).
$$
As before, due to the symmetry between the insider platforms, as well as between the outsiders, we obtain the following optimal post-merger prices:

\[ p^A + C = 24c - 32zc - 19zt - 16t\Lambda + 20z\Lambda + 6t^2 + 14z^2, \]

and

\[ p^B = p^D = 12c - 16zc - 9zt - 4t\Lambda + 4z\Lambda + 3t^2 + 6z^2. \]

Note that: \( p^A + C - c + \Lambda \geq 0 \forall \Lambda \in [0, c] \) but \( p^B - c \geq 0 \) iff \( \Lambda \leq \frac{3}{8}(t - z) \equiv \Lambda. \)

Let now \( g(\Lambda, z, c) \) stand for the post/pre-merger price difference for the insider platforms on either side of the market:

\[ g(\Lambda, z, c) = p^A + C - p^* = \frac{(4t - 5z) (12z\Lambda - 8t\Lambda - zt + 2z^2)}{8 (3z - 2t) (4z - 3t)}. \]

By the same token, the outsider-platform post-/pre-merger price difference on either side of the market writes

\[ l(\Lambda, z, c) = p^B - p^* = \frac{1}{4} \frac{(t - z) (12z\Lambda - 8t\Lambda - zt + 2z^2)}{3z - 2t (4z - 3t)}. \]

Based on the monotonicity of \( g \) and \( l \) (see details in Appendix 2), and restricting the analysis to the interval \( z \in [0, 0.5t) \) satisfying the SOCs, we find that:

**Lemma 2** There exists a threshold of cost savings \( \hat{\Lambda} \) such that the merger does not affect prices.

It is straightforward to check that \( \hat{\Lambda} = \frac{1}{4} \frac{(2z-t)z}{2t-3z} \) solves for \( g(\Lambda, z, c) = l(\Lambda, z, c) = 0 \). Note however that \( \hat{\Lambda} \leq 0 \) for \( z \in [0, 0.5t) \).

**Proposition 3** Consider a bilateral merger between opposite platforms on a perfectly symmetrical four-firm two-sided market; then both the insider and the outsider platforms lower their prices regardless of the amount of cost savings generated by the merger.

Proposition 3 establishes the relationship between the post-merger pricing behavior, the amount of cost savings generated by the merger and the intensity of the indirect externality. Note first that for \( z = 0 \) and \( \Lambda = 0 \), the standard outcome from one-sided spatial oligopoly literature still holds: the merger between distant firms does not affect prices, since the two merging firms have no captive demand between them to exploit. In turn, as soon as the indirect externality is present, the merger’s efficiency gains no longer matter, since in that case the post-merger prices fall anyway. In other words, the distant merger yields an extreme case of
complementarity between the market’s two-sidedness and the merger’s efficiency gains, to the extent that only the first matters for the pricing behavior of platforms. In addition, the type of merger appears to be crucial for the relative importance of the cross-group externality: the latter is clearly dominant for the pricing behavior in the case of a merger between distant platforms.

Turning now to the profitability analysis, this is again relatively straightforward in our perfectly symmetric framework. Profits for insiders and outsiders respectively write now as follows:

\[ \Pi_A + C = N \frac{2}{64} \left( \frac{\Lambda^2 (-16) (3z - 2t)^2 + \Lambda (-8) (3z - 2t) (18t^2 - 43tz + 26z^2)}{(t - z) (4z - 3t)^2} \right) + \Lambda (2z - t) (206z^3 - 144tz^3 - 555t^2z^2 + 492t^2z) \]

and

\[ \Pi_B = \Pi_D = N \frac{2}{64} \left( \frac{\Lambda^2 (-16) (5z - 4t) (3z - 2t) + \Lambda (14z^3 - 18t^3 - 53tz^2 + 56t^2z)}{(t - z) (4z - 3t)^2} \right) \equiv \Pi_O, \]

decreasing the profit differentials equal:

\[ \hat{\Pi}_A + C - 2\Pi^* = \frac{N}{64} \left( \frac{\Lambda^2 (-16) (3z - 2t)^3 + \Lambda (-8) (18t^2 - 43tz + 26z^2) (3z - 2t)^2}{(t - z) (3z - 2t) (4z - 3t)^2} \right) \]

and

\[ \hat{\Pi}^O - \Pi^* = \frac{N}{64} \left( \frac{\Lambda^2 32tN (3z - 2t) (11z - 4t)}{(t - z) (3z - 2t) (4z - 3t)^2} \right) + \Lambda (2z - t) \left( 106z^3 - 72t^3 - 285t^2z^2 + 250t^2z \right) \]

respectively. Then the following obtains:

**Proposition 4** Consider a bilateral merger between distant platforms with symmetric indirect externality; then the merger will always harm outsider platforms but is internally profitable iff it generates enough cost savings, i.e. \( \Lambda \in [\Lambda^*, \Lambda) \).

Proposition 4 establishes the impact of merger cost savings on the internal and external merger profitability.

On the one hand, it extends Brito’s (2005) result to two-sided markets, since our analysis confirms that outsider platforms located next to the merging platforms are harmed by the
merger. This is straightforward to explain: both the indirect externality and the cost savings provide incentives for the insiders to lower their prices. The outsider platforms will do the same, but by less, thus losing demand in the process. The negative impact on their profit is unambiguous, all the more so that our four-firm setting implies that each outsider faces a price drop both to the left and to the right of its location\textsuperscript{13}.

On the other hand, Proposition 4 equally establishes that for the ’distant’ merger to take place, the insider platforms need enough cost savings. Again, this reveals a substitutability-like relationship between the degree of ’two-sidedness’ and the merger efficiency gains, to the extent that the distant-merger market equilibrium is only compatible with a ’weak’ indirect externality: \( z \in [0, 0.5t] \) - in other words, the ’low’ two-sidedness cannot compensate for the standard business stealing effect, therefore it takes high enough cost savings to make the merger profitable. Note moreover that this differs from the seemingly similar result obtained for the adjacent-platform merger with ’strong’ indirect externality. In both cases the merger is internally profitable only if it generates enough cost savings, but this result obtains for different combinations of degree of ’two-sidedness’ and type of merger, and involves different relationships (substitutability or complementarity) between cost savings and indirect externality\textsuperscript{14}.

Finally, the policy implication of Propositions 3 and 4 are quite direct:

\textbf{Corollary 2} \textit{In the case of bilateral merger between distant platforms with symmetrical indirect externalities, the efficiency defense is useless, since this type of merger is pro-competitive whenever submitted.}

One final observation is nevertheless worth making: what really matters for the competitive analysis of mergers on two-sided markets with differentiated platforms is the combination of ’merger type’ and degree of ’two-sidedness’. To see this, consider the case of a relatively weak indirect externality: if the merger involves neighbor firms, meaning closely substitutable platforms, then the efficiency defense is necessary to make sure that customers are not harmed. However, if the merger involves distant, or less substitutable platforms, our analysis indicates that competition agencies can spare the money and time usually spent on efficiency defense arguments, since this merger is pro-competitive.

\textsuperscript{13}This negative profit effect is magnified by the fact that it occurs on both sides of the market as well.

\textsuperscript{14}Note that in the case of merger between neighbor platforms, both ’weak’ and ’strong’ (i.e. \( z \in [0, 0.5t] \cup (0.75t, t) \)) indirect externalities allow for the market equilibrium, but the latter is compatible only with a relatively weak indirect externality (\( z \in (0, 0.5t) \)) in the case of a distant merger.
4 Concluding remarks

This paper studied bilateral horizontal mergers on a two-sided market with four symmetrically differentiated platforms. We provided a theoretical analysis of the merger’s price effect based on the amount of cost savings it generates and the size of the cross-group network effects, assumed to be symmetrical between the two sides of the market. Our main results can be summarized in the following table:

<table>
<thead>
<tr>
<th></th>
<th>weak 'two-sidedness'</th>
<th>strong 'two-sidedness'</th>
</tr>
</thead>
<tbody>
<tr>
<td>adjacent merger</td>
<td>prices ↓ for high cost savings</td>
<td>prices always ↓</td>
</tr>
<tr>
<td></td>
<td>insider II always ↑</td>
<td>insider II ↑ for high cost savings</td>
</tr>
<tr>
<td></td>
<td>outsider II ↓ for high cost savings</td>
<td>outsider II always ↓</td>
</tr>
<tr>
<td>distant merger</td>
<td>prices always ↓</td>
<td>no interior equilibrium</td>
</tr>
<tr>
<td></td>
<td>insider II ↑ for high cost savings</td>
<td></td>
</tr>
<tr>
<td></td>
<td>outsider II always ↓</td>
<td></td>
</tr>
</tbody>
</table>

We also discussed the merger control policy implications: the market’s ‘two-sidedness’ makes the efficiency defense redundant if the merger takes place between distant platforms. In the case of adjacent merging platforms, however, the efficiency defense is still necessary if the cross-group externality is relatively weak.

Our analysis can be extended to other cases, such as different market sizes on each side of the platforms, or asymmetric indirect externalities (only one side benefitting from the presence of the other), and we leave this for future research.

References


5 Appendix

5.1 Appendix 1: Merger between adjacent platforms

Hessian matrices

Whenever the SOCs are satisfied the Hessian matrices for all profit functions are negative definite (ND). We define below the Hessian matrix $H^{A+B}$ for the merged firm and $H^{C,D}$ for the outsider profits:

\[
H^{A+B} = \frac{N}{t^2-4z^2} \begin{bmatrix}
\frac{t(5z^2-2t^2)}{(t^2-z^2)} & \frac{2(2z^2-t^2)}{(t^2-z^2)} & t & 2z \\
\frac{2z}{(t^2-z^2)} & t & 2z & t \\
2z & t & \frac{3z(2z^2-t^2)}{(t^2-z^2)} & \frac{3z(2z^2-t^2)}{(t^2-z^2)} \\
\end{bmatrix}
\]

and

\[
H^{C,D} = \frac{N}{(t^2-z^2)(t^2-4z^2)} \begin{bmatrix}
\frac{t(5z^2-2t^2)}{(t^2-z^2)} & \frac{3z(2z^2-t^2)}{(t^2-z^2)} & t & 2z \\
\frac{3z(2z^2-t^2)}{(t^2-z^2)} & t & \frac{3z(2z^2-t^2)}{(t^2-z^2)} & \frac{3z(2z^2-t^2)}{(t^2-z^2)} \\
\end{bmatrix}
\]
In order for $H^{A+B}$ and $H^{C,D}$ to be ND the following principal minor conditions need to be fulfilled:

for the merged firm: $|H_1^{A+B}| = \frac{Nt(-2t^2+5z^2)}{4z^2+4t^2-5z^2t^2} < 0$, $|H_2^{A+B}| = -\frac{N^2(9z^2-4t^2)}{4z^2+4t^2-5z^2t^2} > 0$, $|H_3^{A+B}| = \frac{N^3(11z^2-6t^2)}{4z^2+4t^2-5z^2t^2} < 0$;

- for each outsider: $|H_1^{C,D}| = \frac{Nt(-2t^2+5z^2)}{4z^2+4t^2-5z^2t^2} < 0$, $|H_2^{C,D}| = -\frac{N^2(9z^2-4t^2)}{4z^2+4t^2-5z^2t^2} > 0$.

One can relatively easily check that the SOCs are satisfied for $z < 0.5t$ or $0.75t < z < t$.

**Proof of Lemma 1 & Proposition 1**

Recall that the price differences for the insiders ($A$ and $B$) and the outsiders ($C$ and $D$) respectively write $f(\Delta, z, c) = \tilde{p}^{A+B} - p^* = \frac{3(3-5z)(2+6z)\Delta-2t(4\Delta+t^2)}{2(5t-8z)(2t-3z)}$ and $h(\Delta, z, c) = \tilde{p}^C - p^* = \frac{1}{2(2t-3z)(5t-8z)}(6z\Delta - 4t\Delta - 2st + t^2 + z^2)$, with $f(\Delta, z, c) = h(\Delta, z, c) = 0$ for $\Delta = -\frac{z^2}{2(2t-3z)}$.

One has that $\frac{\partial}{\partial \Delta} f(\Delta, z, c) = \frac{3-5z}{8z-5t} < 0$ for $z \in [0, 0.5t) \cup (0.75t, t)$ and $\frac{\partial}{\partial \Delta} h(\Delta, z, c) = \frac{2t-3z}{8z-5t} < 0$ for $z \in [0, 0.5t) \cup (0.75t, t)$ as well. Note also that for $\Delta = 0$ the price differences write $f(0, z, c) = \frac{3(3-5z)(z-t)^2}{2(2t-3z)(5t-8z)}$ and $h(0, z, c) = \frac{(t-z)(z-t)^2}{2(2t-3z)(5t-8z)}$. Therefore $f(0, z, c) < 0$ and $h(0, z, c) < 0$ if $z \in (0.75t, t)$, but $f(0, z, c) \leq 0$ and $h(0, z, c) \leq 0$ when $\Delta \geq \tilde{\Delta}$ if $z \in [0, 0.5t)$.

**Proof of Proposition 2**

- The merger profitability for the insider platforms is measured by $\tilde{\Pi}^{A+B} - 2\Pi^*$, where:
  
  $\tilde{\Pi}^{A+B} - 2\Pi^* = N \frac{4(2t-3z)^3\Delta^2+4(2t-7z)(t-z)(2t-3z)^2\Delta+(19z^2-23tz+7t^2)(t-z)^3}{2(t-z)(8z-5t)^2(2t-3z)}$

  We have that $\tilde{\Pi}^{A+B} - 2\Pi^* = 0$ if $\Delta = \tilde{\delta}_1$ or $\Delta = \tilde{\delta}_2$, where:
  
  $\tilde{\delta}_1 = (z-t) \frac{(4t-7z)(2t-3z)+\sqrt{(2z-t)(3z-2t)(8z-5t)^2}}{2(3t-2t)^2}$

  and $\tilde{\delta}_2 = (z-t) \frac{(4t-7z)(2t-3z)-\sqrt{(2z-t)(3z-2t)(8z-5t)^2}}{2(3t-2t)^2}$.

  We find that:
  
  - if $z \in [0, 0.5t)$, $\tilde{\delta}_1 < 0$ and $\tilde{\delta}_2 < 0$, that is $\tilde{\Pi}^{A+B} - 2\Pi^* > 0$.
  - if $z \in (0.75t, t)$, $0 < \tilde{\delta}_2 < \Delta$ and $\tilde{\delta}_1$ remains negative, that is $\tilde{\Pi}^{A+B} - 2\Pi^* > 0$ if $\Delta > \tilde{\delta}_2 \equiv \Delta^*$.

- The merger’s impact on the outsiders’ profits is measured by $\tilde{\Pi}^O - \Pi^*$, where: $\tilde{\Pi}^O - \Pi^* = N \frac{(t-2z)(4t-6z)^2\Delta^2-12(t-z)(3z-2t)^2\Delta+(t-z)^3(11t-17z)}{4(t-z)(2t-3z)(8z-5t)^2}$
We have that $\tilde{\Pi}^{C,D} - \Pi^* = 0$ if $\Delta = \delta_1^O$ or $\Delta = \delta_2^O$, where $\delta_1^O = \frac{(t-z)^2\Delta}{2(2t-3z)}$ and $\delta_2^O = \frac{(11t-17z)(t-z)}{2(2t-3z)}$.

We conclude that:
- if $z \in [0, t/2)$, $0 < \delta_1^O < \Delta < \delta_2^O$ and then $\tilde{\Pi}^O - \Pi^* \geq 0$ when $\Delta \leq \tilde{\Delta}$ and $\tilde{\Pi}^O - \Pi^* \leq 0$ when $\tilde{\Delta} \leq \Delta \leq \bar{\Delta}$;
- if $z \in (3t/4, t)$, $0 > \Delta > \delta_2^O$ and then $\tilde{\Pi}^O - \Pi^* \leq 0$ when $\Delta \leq \bar{\Delta}$.

### 5.2 Appendix 2: Merger between distant platforms

**Hessian Matrices:**
- for the merged firm $(A + C)$:

$$
H^{A+C} = \frac{N}{(t^2-z^2)(t^2-4z^2)} \begin{bmatrix}
3z(2z^2 - t^2) & -t(-2t^2 + 5z^2) & -3z(2z^2 - t^2) & -z(2t^2 + 5z^2) \\
3z(2z^2 - t^2) & t(-2t^2 + 5z^2) & -z(t^2 + 2z^2) & -3z^2t \\
-3z^2t & -z(t^2 + 2z^2) & t(-2t^2 + 5z^2) & 3z(2z^2 - t^2) \\
-z(t^2 + 2z^2) & -3z^2t & 3z(2z^2 - t^2) & t(-2t^2 + 5z^2)
\end{bmatrix}
$$

- and for the outsiders $(B$ or $D)$:

$$
H^O = \frac{N}{(t^2-z^2)(t^2-4z^2)} \begin{bmatrix}
3z(2z^2 - t^2) & t(-2t^2 + 5z^2) \\
3z(2z^2 - t^2) & t(-2t^2 + 5z^2)
\end{bmatrix}
$$

Leading to principal minors and conditions:
- for the insider platforms: $|H^{A+C}_1| = -\frac{N(2t^2-5z^2)}{5z^2t^2+4z^2+t^2} < 0$, $|H^{A+C}_2| = \frac{N^2(4t^2-9z^2)}{5z^2t^2+4z^2+t^2} > 0$;

- for the outsiders: $|H^{B,D}_1| = -\frac{N(2t^2-5z^2)}{5z^2t^2+4z^2+t^2} < 0$, $|H^{B,D}_2| = \frac{N^2(4t^2-9z^2)}{5z^2t^2+4z^2+t^2} > 0$.

The SOCs are satisfied for $z < 0.5t$.

### Proof of Lemma 2 & Proposition 3

Recall that the price differences for the insiders $(A$ and $C)$ and the outsiders $(B$ and $D)$ respectively are given by $g(\Lambda, z, c) = \hat{p}^{A+C} - p^* = \frac{(4t-5z)(2z^2+12z\Lambda-z^2-8t\Lambda)}{8(3t-4z)(2t-3z)}$ and $l(\Lambda, z, c) = \hat{p}^{B,D} - p^* = \frac{(t-z)(2z^2+12z\Lambda-z^2-8t\Lambda)}{4(2t-3z)(3t-4z)}$.

Recall also that $g(\Lambda, z, c) = l(\Lambda, z, c) = 0$ for $\Lambda = \frac{z(2z-t)}{4(3t-2z)} < 0$, where $\Lambda < 0$ for $z \in [0, 0.5t)$. Moreover, $\frac{\partial}{\partial \Lambda} g(\Lambda, z, c) = \frac{4t-5z}{8z^2}$ and $\frac{\partial}{\partial \Lambda} l(\Lambda, z, c) = \frac{t-z}{4z^2}$, therefore both price differences are decreasing in $\Lambda$ for $z \in [0, 0.5t)$. Therefore one has that for all $z \in [0, 0.5t)$, $g < 0$ and $l < 0$ as well.
Proof of Proposition 4:

- The profitability for the merging platforms is given by $\hat{\Pi}^{A+C} - 2\Pi^*$, where:

\[
\hat{\Pi}^{A+C} - 2\Pi^* = -N \frac{16(2t-3z)^3\lambda^2 - 8(-43tz^2 + 18t^2 + 26z^2)(2t-3z)^2\lambda + z(t-2z)(285tz^2 - 250t^2z + 72t^3 - 106z^3)}{64(4z-3t)^2(z-t)(3z-2t)}
\]

We find that $\hat{\Pi}^{A+C} - 2\Pi^* = 0$ if $\Lambda = \lambda_1$ or $\Lambda = \lambda_2$, where

$\lambda_1 = (2t-3z)(26z^2 - 43zt + 18t^2) + 2 \sqrt{(t-z)(2t-3z)(35z^2 - 49zt + 18t^2)(-4z + 3t)}$, and

$\lambda_2 = (2t-3z)(26z^2 - 43zt + 18t^2) - 2 \sqrt{(t-z)(2t-3z)(35z^2 - 49zt + 18t^2)(-4z + 3t)}$.

One can easily check that for $z < 0.5t$, $\lambda_1 > \lambda_2 > 0$ and $\lambda_1 > \Lambda$. We check that $\hat{\Pi}^{A+C} - 2\Pi^* \geq 0$ if $\Lambda \geq \lambda_2 \equiv \Lambda^*$.

- The merger’s impact on the outsiders’ profits is measured by $\hat{\Pi}^O - \Pi^*$, where:

\[
\hat{\Pi}^O - \Pi^* = -N \frac{16(4t-5z)(2t-3z)^2\lambda^2 + 8(-2t + 3z)(14z^3 - 18z^3 + 56zt^2 - 53z^2t)\lambda - z(t-2z)(38z^3 - 36z^3 + 116zt^2 - 119z^2t)}{64(4z-3t)^2(z-t)(3z-2t)}
\]

One has that $\hat{\Pi}^{B,D} - \Pi^* = 0$ if $\Lambda = \lambda_1^O$ or $\Lambda = \lambda_2^O$, where $\lambda_1^O = -z(2z-t) \equiv \hat{\Lambda} < 0$, and

\[\lambda_2^O = \frac{38z^3 - 119z^2t + 116zt^2 - 36z^3}{(2t-3z)(4z-5z)} < 0 \text{ also for } z \in [0, 0.5t].\]

One can easily check that $\hat{\Pi}^O - \Pi^* < 0$ for all $z \in [0, 0.5t)$.