The Paradox of Legal Unification*

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Abstract

Countries often choose collective approaches resulting in the unification of the legal rules of individual countries. However, in simple standard games of legal harmonization, standard cooperative approaches of analysis are unable to reproduce any collective choice leading to legal unification. We call this dissonance the paradox of unification. We study if some modifications in assumptions about preferences or cooperative solution concepts can solve this paradox. While the introduction of social preferences or of a Kantian concept of equilibrium doesn’t resolve the paradox, legal unification can be a Berge equilibrium of this legal standardization game.

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1 Introduction

1.1 Legal Unification: terminology and examples

Legal unification can be defined as the substitution of multiple national rules by a new and unique legislation for all countries choosing to participate to the process. The objective is to render uniform the legal responses of different countries facing the same facts or situations, irrespective of the place they occur or of the national elements involved. The way countries promote legal unification can take different forms. In the European Union, where uniformity of legal decisions is an important stake, unification of laws is driven using the tool of regulations, which are European legal acts immediately enforceable as law in all member states simultaneously. The aim of regulations differs clearly from the EU’s other main tool, the Directives, which need to be transposed into national laws and can then differ among countries. This latter case is referred to as Legal Harmonization. For some laws, concerning for example fundamental Human Rights, European countries impose complete uniformity, including for new members. For others parts of the law, harmonization is preferred (Boele-Woelkin, 2010).

Another possible tool aiming to unify laws is the international Convention. Boele-Woelkin (2010) argues for example that "the unification of substantive private law is predominantly achieved by international Conventions". A Convention does not create per se unified or uniform law: unification remains an individual action for States, since it requires an individual legislative action. Unification is only achieved after the ratification of the unifying instrument. Unification is then a top-down approach, which imply at least some part of collective decision-making, but without the disappearance of individual decisions from countries.

An example of international legal unification by a Convention is the unilateral ratification by several countries of the United Nations Convention on Contracts for the International Sale of Goods (CISG) many years after the finalisation of the text, written in 1980 during a diplomatic conference representing 62 states. The aim was the promotion of international trade through the adoption of uniform rules. The drafters of the Convention were explicitly stating that in the application of the Convention, uniformity was needed (Ferrari, 1996). The international character of the law means none national law should be used to interpret the Convention, which has to be analyzed as international and independent law. In this

\[\text{Article 7(1) of the Convention: "In the interpretation of this Convention, regard is to be had to its international character and to the need to promote uniformity in its application and the observance of good faith in international trade."}\]
sense, legal unification often means the disappearance of references to national laws.

The trend toward uniformization of laws is an active topic in the legal literature, since diversity of national legal rules is seen as an inefficient institution in an era of rising international trade. The advocacy of legal unification is not recent and for example the legal comparatist David (1968) was early arguing that "the problem is not whether international unification of law will be achieved; it is how it can be achieved". From a law viewpoint, the question is to understand which tools and protocols of decision are the best suited to realize legal uniformity. From an economics viewpoint, the question means to understand under which conditions unification of laws can be an equilibrium.

When choosing the appropriate economic framework to analyze these questions, it is important to note that legal unification is both a result and a process. It is a result in the sense that all legal rules become the same for countries choosing to unify. It is also a process in the sense that unification results from some kind of collective decision. There is some debate on the possibility that legal unification also occurs from purely non cooperative interactions between countries. Herings and Kanning (2008) argue for example that the CISG can be considered as a non cooperative process, since countries chose individually to ratify the Convention. To distinguish the non cooperative approach from ours, we call legal uniformization an outcome in which all countries choose the same laws, without any reference to the process used to reach it. Legal unification requires both legal uniformization and some type of collective decision making process. Since the CISG was designed during a Convention, it seems difficult to overshadow this collective aspect in construction of legal uniformization, even if countries were later free to ratify the Convention or not. It is worth noting that all collective decision processes do not lead to unification, since countries often aim to obtain only some harmonization of legal rules. Each outcome has specific advantages and disadvantages (Boele-Woelkin, 2010).

1.2 The Law and Economics of Legal Unification

The law-and-economics literature on legal change study these questions building its analysis on the assumption of a fundamental trade-off. On one hand, legal convergence decreases the cost of legal diversity. On the other hand legal convergence increases the cost of the discrepancy between national laws and national preferences. Given this tradeoff, a paradox arises: while observed in reality, legal uniformity is never an equilibrium outcome of a standard legal standardization game, even when countries play cooperatively. We call
this inability of standard model to predict unification the *paradox of legal unification*. The inability of cooperative methods to rationalize unification results from the way legal cooperation is modeled. Legal cooperation is construed as the maximization of a sum of the countries objective function (where the choice variables are the national legal systems). Legal unification is not a solution of this problem.

Since legal unification cannot be rationalized from standard cooperative approaches, it can be interesting to understand which parts of the problem can be the source of this paradox. First, we can see if alternative assumptions on preferences can solve this paradox. Modifying assumptions on preferences has became a standard way to resolve paradoxes put forward by the experimental research on social dilemmas. The change in preferences often consists in the introduction of other-regarding preferences: each player takes at least partly into account the welfare of other agents in its own objective function. Implementing such modifications of preferences is not trivial since the appropriate modification is context-dependent, in a non obvious way. We then study a general formulation of other-regarding preferences and we show that such a modification doesn’t solve the paradox of legal unification.

Another approach can consist in modifying the assumed protocols of decisions used by the players of the game, rather than their preferences. Two motivations can lead to the use of these alternative solution concepts. The first one is the inability of standard approaches to rationalize some outcomes, such as legal unification in our case. The second motivation relies on the idea that changing protocols of decision could be preferred to changing preferences in order to reproduce specific outcomes. One may argues that it seems more appropriate to think that the protocol of choice of agents can be flexible and context-dependent than preferences, which are often considered as ”deep parameters”, even if some changes can occur in the long run.

The recent literature has proposed different concepts of solution to analyze the choices of socially-oriented agents. Among them, an important one relies on Kantian rules of behavior. When using Kantian rules, agents have to make a choice such that, from their viewpoint, this choice should become a universal law (i.e., it should be followed by other agents). This constraint of behavior is not externally imposed and must result from the choice of each individual, that he has set for himself. Discussing the financing of public expenditures, Samuelson (1954) was already citing this ”Kant’s categorical imperative” as a possible scheme to attain alternative and preferred social outcomes. Laffont (1975) was the first to propose an economic modeling of Kantian rules of choices in a game theoretic setup. More recently, Roemer (2015) has proposed alternative approaches to model Kantian choices (see
also Roemer, 2010). It is important to note that while this rule of behavior has some social orientation, it remains fully individualistic.

Another approach of the literature on socially oriented behavior builds on the idea that players can use some kind of team reasoning and can notably choose to support each other in games. Several concepts can be associated to team reasoning. Among them, the concept of Berge equilibrium models explicitly an idea of mutual support in games, by introducing some altruistic social value orientation in the way agents play games. In a Berge equilibrium (Berge, 1957, Zhukovskii and Chikrii, 1994, Colman et al. (2011)), each player supports his teammates (the other players) and is supported by them in return. Formally, it means that all the other players maximize his payoff, while himself maximizes the payment of the other players.

The last socially oriented concept of solution we study builds around universal ethical concepts, which influence the behavior of agents toward socially inclined goals. The concept we study is often named as ”Golden Rules”. As stated by Singer (1993), ”the major ethical traditions all accept, in some form or other, a version of the Golden Rule that encourages equal consideration of interests. ”Love your neighbour as yourself, said Jesus”. ”What is hateful to you do not do to your neighbour’, said Rabbi Hillel. Confucius summed up his teaching in very similar terms: ”What you do not want done to yourself, do not do to others”. The Mahabharata, the great Indian epic, says: ”Let no man do to another that which would be repugnant to himself”. In a recent paper, Van Damme (2014) formally analyzes the following version of the Golden Rule: ”Do unto others as you like others to do unto you”). Indeed, this concept is highly related to the previous concept of Berge equilibrium.

In this paper, we apply all these different socially-oriented concepts of solution to a standard legal standardization game. Our aim is to study if one of these variations is able to predict to rationalize legal unification as an equilibrium outcome, giving an additional tool to analyse some social interactions.

The remaining part of the papers unfolds as follows. In the next section we lay out a general legal standardization game. We then present the paradox of legal unification in section 3. We study alternative solution concepts in section 4. Section 5 contains some concluding remarks. All the proofs are relegated to the appendix.
2 A Legal Standardization Game

In this section we first lay out a general model of legal standardization. Then, we state what we call the paradox of legal unification. Finally, we show how this paradox has been addressed in the law-and-economics literature.

2.1 The Model

We consider a game with a set of players composed by $N$ countries. Country $i$ chooses (or adapts) its own legal system $x_i$, in order to maximize its utility function $U_i$. We assume that

$$U^i(X) = U^i(x_i, \theta_i) + V^i(x_i, X_{-i}),$$

(1)

where $X$ is the $N$-vector of the countries choices, $\theta_i$ is a parameter which describes the culturally ideal law of country $i$ (e.g., its legal preferences), and $X_{-i}$ is the $N-1$ vector of all the countries choices but country $i$’s. We assume that the culturally ideal laws $\theta_i$ are all different.$^2$

We assume that both the actual and the culturally ideal laws can be associated to points of the real line. We interpret these points as aggregate indexes of legal rules concerning a specific issue of the legal system.$^3$

Further, we assume that the function $U^i$ is single-peaked with respect to $\theta_i$ and that the function $V^i$, when viewed as a function of $x_k$, $k \neq i$, is single-peaked with respect to $x_i$. The interpretation of these assumption is straightforward. Utility of country $i$ rises when its legal system is closer to its legal preferences and also when the foreign legal systems are also closer to its own legal system.$^4$

$^2$ This class of games is not specific to Law and Economics. It can be found for example in Macroeconomics, since individual Lucas-Phelps supply functions have the same form Myatt and Wallace (2014). Another example is the Keynes’ beauty contest or the analysis of behavior in presence of social norms Bicchieri (2006). However, there is less focus in the questions on equilibria in which all agents make the same choice.

$^3$ The construction of aggregate indexes of legal rules is a current practice in the empirical law-and-economics literature. For instance, the Leximetrics database comprises several quantitative indexes of legal rules about corporate governance, creditor law or labour law (see siems2011measuring).

$^4$ These distance reflect transaction costs in a broad sense. These costs are often highlighted in the legal literature (Linarelli, 2002). Rodrik (2004) argues that the diversity of national institutional arrangements is the most important source of transaction costs in international exchanges, broadly representing nearly 35% in ad-valorem terms. Wagner (2005) also displays the different costs associated to the specific legal uncertainty when a firm engages in cross-border transactions. The existence of these costs call more legal
We shall also assume that the functions $U^i$ and $V^i$ are smooth. In addition, we suppose that

$$\frac{\partial V^j}{\partial x_i} \bigg|_{x_k=x, k=1,\ldots,n} = 0, \forall j, \forall i. \quad (2)$$

The above equation means that there is no interest in changing $x_i$ if legal unification is achieved. The following specification of the utility function satisfies our assumptions

**Example**

Loeper (2011) proposes the following example:

$$U^i(X) = -\frac{1}{2}(x_i - \theta_i)^2 - \frac{\beta}{2N} \sum_{j \neq i} (x_i - x_j)^2. \quad (3)$$

Another example of a utility function that satisfies all of our assumption is

$$U^i(X) = -\sqrt{a + (x_i - \theta_i)^2} - \beta \left( \sum_{j \neq i} \sqrt{a + (x_i - x_j)^2} \right), \quad (4)$$

where $a$ is a positive parameter.

The next specification, however, does not satisfy condition (2) if $\mu \neq 1$

$$U^i(X) = -\frac{1}{2}(x_i - \theta_i)^2 + \beta \left( \sum_{j \neq i} \left( x_ix_j - \frac{x_j^2}{2} - \frac{\mu x_i^2}{2} \right) \right). \quad (5)$$

### 3 The paradox of legal unification

The standard cooperative approach to analyze social outcomes do never predict unification as a possible outcome of the previous legal harmonization game. The only way to obtain unification proposed in the literature is to impose *ex ante* a unified choices for countries, and then to find the best possible choice. Unification is then an assumption, not a choice. We call this the Paradox of legal unification.
3.1 The Standard Cooperative Approach

In the literature, the cooperative approach is often associated to the maximization of a weighted sum of the countries objective functions (see, e.g., Carbonara and Parisi, 2007). We thus consider the following problem:

$$\max_X \sum_{i=1}^{N} \beta_i \left( U^i(x_i, \theta_i) + V^i(x_i, X_{-i}) \right),$$

(6)

where the parameter $\beta_i$ corresponds to the weight given to country $i$ in the problem. We assume that all the parameters are non nil. We have the following Proposition:

**Proposition 1.** Legal unification is never a solution to the problem 6.

Legal cooperation, modeled as in problem (6), never leads to legal unification. The intuition of the above Proposition is the following. When the countries laws are chosen so as to solve problem (6), each country chooses its law to satisfy the following first-order optimality condition:

$$\beta_i \frac{\partial U_i}{\partial x_i} + \sum_j \beta_j \frac{\partial V_j}{\partial x_i} = 0$$

(7)

When $x_i$ gets marginally closer to $\theta_i$, the increase in the utility function of country $i$ is compensated by the marginal changes in the utility of the countries (some of these changes may be positive, as $x_i$ gets also closer to $x_j$, but this may not be the case for all countries). In the event that legal unification is realized in a solution to problem (6), all the terms of the sum in the above equation are nil. But this would imply that $x_i = \theta_i$ (since $U^i$ is single-peaked in $\theta_i$). As the $\theta_i$ are by assumption all different this is impossible. We illustrate the Proposition with the next example.

**Example** (quadratic)

Let us consider the problem

$$\max_{(x_i)} \sum_{i=1}^{N} U_i = \max_{(x_i)} \left( \frac{1}{2} \sum_{i=1}^{N} (x_i - \theta_i)^2 - \frac{\beta}{2N} \sum_{i=1}^{N} \sum_{j \neq i} (x_i - x_j)^2 \right).$$
The solution of this problem is given as follows

\[ x_i = \frac{\theta_i + 2\beta \hat{\theta}}{1 + 2\beta}, \quad i = 1, \ldots, N, \]

where \( \hat{\theta} = \left( \sum_{i=1}^{N} \theta_i \right) / N \). We notice that legal unification occurs if and only if \( \theta_i = \theta, \forall i \).

### 3.2 Legal Unification in the Law and Economics Literature

The standard solution to obtain legal unification (see, e.g., Loeper, 2011, Crettez, Deffains, and Musy, 2016) is determined by solving problem (6) under the constraint that \( x_i = x, \forall i \).

\[
\max_{\bar{x}} \sum_{i=1}^{N} U^i = \max_{\bar{x}} \sum_{i=1}^{N} U^i(x_i, \theta_i)
\]

The solution then consists to find the best uniform law. Legal uniformity is an assumption and not a result.

**Example** (quadratic)

In the example considered so far, imposing legal unification implies that all the terms \( V^j \) vanish. Choosing the optimal level of legal unification amounts to solve the following problem

\[
\max_{\bar{x}} \sum_{i=1}^{N} U_i = \max_{\bar{x}} -\frac{1}{2} \sum_{i=1}^{N} (\bar{x} - \theta_i)^2.
\]

The solution of this problem is \( \bar{x} = \hat{\theta} \).

This solution is short of being compelling, since it does not explain unification. It would be more convincing if legal unification were the non-constrained solution of collective choice problem. Given the previous propositions, it seems that to obtain this result, we need to modify the terms of the problem. We address this issue in the next section.

### 4 Alternative Socially Oriented Solution Concepts

A growing body of empirical and theoretical literature argues that economic agents do have social concerns when they play games. Such concerns can be linked to other-regarding
preferences, moral imperatives, collective reasoning or ethical concerns. In this section, we study four of these alternatives. The first one builds on the traditional way to resolve social paradox, by assuming the standard Nash behavior from agents, but by introducing other-regarding preferences. The second concept we study builds on Kantian rules of behavior (Roemer, 2015). The third one, named as the Berge equilibrium, is a possible concept of mutual support (berge57). The fourth one builds one traditional ethics and is named as Golden rules (Van Damme, 2014). The last three alternatives assume standard preferences, but alternative protocols of decision.

### 4.1 Nash equilibrium with other regarding preferences

One may argue that the collective decision making present in Conventions or in the European decision making process do not alter the standard assumptions on the behavior of agents, but that it only imposes on countries to take into account the impact of their choices on the welfare of other countries. Said differently, the correct tool of analysis would still be the Nash equilibrium, assuming some type of social preferences. The introduction of other-regarding preferences in the utility functions of otherwise strategic agents is standard in the resolution of social paradox in economics. As summarized by Roemer (2015):

> Economic theory, for the main, attempts to explain cooperative behavior as the non-cooperative equilibrium of a complex game with many stages. The innovation of behavioral economics is to include exotic arguments in preferences (for example, a sense of fairness) but the analytical structure is still Nash (non-cooperative) equilibrium.

Using this approach (in the line of Roemer, 2015), the question is the following: "Is it possible to define a rule for transforming the utility function $\mathcal{U}^i$ into a utility function with more arguments $\mathcal{T}^i$, such that the Nash equilibrium of the extended environment implies legal unification?"

To study this point, we assume that all the additional arguments in the function $\mathcal{T}^i$ come from the utility functions of the other players $\mathcal{U}^{-i}$. With such modifications, the utility function of any player $i$ can be written as follows:

$$\mathcal{T}^i \left( \mathcal{U}^i(x_1, X_{-1}), ..., \mathcal{U}^i(x_i, X_{-i}), ..., \mathcal{U}^N(x_N, X_{-N}) \right) \quad (8)$$

with $\mathcal{U}^i(X) = U^i(x_i, \theta_i) + V^i(x_i, X_{-i})$ and $\mathcal{U}^j(X) = U^j(x_j, \theta_j) + V^i(x_j, X_{-j}), \forall j \neq i$. 

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Given these modified utility functions, we ask whether some functions $T^i$, $i = 1, ..., N$ defined on $\mathbb{R}^N$, which are both differentiable and increasing with respect to their arguments, can reproduce legal unification.

As the following Proposition shows, the answer to this question is negative

**Proposition 2.** There are no differentiable-increasing functions, $T^i$ defined on $\mathbb{R}^N$ such legal unification is realized in at least one Nash equilibrium of the game where the countries’ objective functions are given by the function $T^i(U^i, U^{-i})$, $i = 1, ..., N$.

Even in the presence of other-regarding preferences, it remains always beneficial for at least one country to deviate from a unique legal rule, in order to be closer to its own legal preference.

### 4.2 Kant-Roemer Equilibrium

An alternative solution concept with a social orientation is the Kant equilibrium, first modeled by Laffont (1975). This equilibrium is named after Kant, since for this philosopher, there is a form of rationality which generates categorical imperatives about behavior (Do $X$, regardless of your wants). A choice is rational if it is prescribed by some principle in which the chooser can will that his action becomes a universal law for all rational agent. Categorical imperatives are dictated by reason alone (not passions). An autonomous agent imposes his own laws; but if each agent arrives at these laws by the use of reason, all will arrive at the same law (the law must be become universal). In the game of legal harmonization we study, there is no Kant-Laffont equilibrium when the ideal legal laws are different across countries. However, Roemer (2015) has recently proposed another interpretation of the Kantian imperative. The characteristic of the Kantian approach, as Roemer states, is "not to ask individuals to put themselves in other people’s shoes, but rather to evaluate how they would fare if all other behaved as they do".

Roemer has proposed several definitions (see, *e.g.*, Roemer, 2015). We focus on one definition, the notion of *additive Kantian equilibrium*.\(^5\) We shall say that a vector of strategies $X^K$ is an additive Kantian equilibrium, if, given a vector $A = (a_1, \ldots, a_i, \ldots, a_n) \in \mathbb{R}^n_{++}$,

\(^5\) While this notion is suited for non-symmetric model, Roemer restricts his definition to the case where agents have the same strategy space, namely a subset of non-negative real numbers. Moreover, Roemer mainly considers the case, where each agent’s objective is either monotonically increasing or monotonically decreasing with respect to the others’ strategies. In our model, this would only be the case if all the countries decisions were either lower than $\theta$ or greater than $\bar{\theta}$. 11
we have\(^6\)

\[
\forall i, \quad 0 = \arg \max_{\alpha \in \mathbb{R}} U^i(x_i^K + \alpha a_i, \theta_i) + V^i(X^K + \alpha A).
\]  

(9)

We have the following result

**Proposition 3.** Let \(X^K\) be an additive Kantian equilibrium. Then legal unification is not achieved.

The individualistic nature of the Kantian equilibrium prevent players to make the same choice.

### 4.3 Berge equilibrium

We introduce another definition of cooperation, using the notion of Berge equilibrium, presented in Colman et al. (2011) and Courtois, Nessah, and Tazdaït (2015). A Berge equilibrium for our legal standardization game is a strategy profile \(X^*\) which satisfies the following inequation\(^7\)

\[
U^i(x_i^*, X_{-i}) \leq U^i(X^*), \quad \text{for all } X_{-i}, \quad \text{for all } i \in N.
\]  

(10)

In a Berge equilibrium a player supports others by choosing an action that maximizes their utilities, and others support him in the same way. This makes Berge equilibrium a *mutual support equilibrium*. The Berge equilibrium formalizes the motto “One for all, and all for one” (also inverted to “All for one, and one for all”) traditionally associated with the King’s Musketeers in the novel *The Three Musketeers* written by Alexandre Dumas.

The concept Berge equilibrium is an instance of Team reasoning theory. This theory has been developed to overcome the predictive failures of standard game theory in cooperation and coordination games. Here, we contend that countries can conceive themselves as the members of a team, and make their choices so as to satisfy the team’s objective. Team reasoning implies a transfer of agency from the countries to the collective level (on team reasoning and team preferences see, e.g., Sugden (2000)).

\(^6\) See Roemer (2010), p. 6., or Roemer (2015), definition 4, with \(A = \mathcal{X}\).

\(^7\) We follow the definition used in Courtois et al. (2015). A Berge equilibrium is sometimes called a Berge-Zhukovskii equilibrium.
In the context of our legal standardization game the cooperative behavior implied in a Berge equilibrium may reflect a kind of political unity obtained through a certain convergence of political preferences.\(^8\)

A remarkable property of the Berge equilibria of the legal standardization game is that it solves the legal unification paradox. We indeed have

**Proposition 4.** *Legal unification is realized in any Berge equilibrium of the legal standardization game.*

For any Berge equilibrium \(X^*\), there is a real number \(x\) such that \(x^*_i = x\) for all \(i\). This property hinges on the assumption that all the functions \(V^i\) are single-peaked in \(x_i\). In a Berge equilibrium, all countries \(j\) choose \(x_i\) in order to maximize the utility of country \(i\). It follows immediately that legal unification must be realized (since the property is satisfied for all countries).

The next Proposition shows that any real number \(x\) is a Berge equilibrium of the legal standardization game.

**Proposition 5.** Let \(x \in \mathbb{R}\) be given. Then \(x\) is a Berge equilibrium for the legal standardization game.

We may argue that there are too many Berge equilibria to our legal standardization game. However, it is easy to see that all Berge equilibria with \(x\) outside the interval \([\theta, \bar{\theta}]\) are not Pareto-optimal, and that all countries would benefit from a shift from \(x\) to \(\theta\), if \(x < \theta\), or from \(x\) to \(\bar{\theta}\), if \(\bar{\theta} < x\).

Inspecting equation (3) we readily see that in a Berge equilibrium \(x_j = x_i\) for all \(i \neq j\). Conversely, we also see that if \(x_i = x\), then to maximize \(U^i\) we must have \(x_j = x\). This is true for all \(i\) and which proves that \(x\) is Berge equilibrium.

The Kantian approach doesn’t require an individual to be empathetic, in the sense of taking the preferences of other people into account. It requires in return individual to take "similar actions". On the contrary, the Berge approach requires an individual to take the preferences of other people into account, but not necessarily to take "similar actions". Paradoxically, it leads to unification in the latter case but not in the former.

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\(^8\) On the convergences of political preferences, see Goodman and Jinks (2013) on States socialization and, in a different vein Katzenelson and Weingast (2005).
4.4 Golden Rule Equilibrium

The last concept we consider builds on universal ethical concepts. More specifically, we consider rules of behavior that can be summarized under the name of Golden Rules. A formal analysis of the notion of golden rule has been recently proposed by Van Damme (2014), using the following version: "Do unto others as you like others to do unto you". This is very similar to the notion of mutual support in Berge equilibria, since what you would like other do unto you is to maximize your utility, then in return, what you have to do to them is to maximize their utility.

In our setup, a weak golden rule is a decision \( r \) in \( \mathbb{R} \) such that for all \( i \)

\[
U^i(r \mathbb{1}) = \max_{X_{-i}} \{ U^i(r, \theta_i) + V^i(r, X_{-i}) \}
\]

(11)

where \( \mathbb{1} \) is the vector of \( \mathbb{R}^n \) whose components are all equal to 1.\(^9\)

We can see that a weak golden rule is thus a Berge equilibrium of the legal harmonization game. Actually, all real number \( r \) is a weak golden rule for the this game. Generally speaking a Berge equilibrium is not necessarily a golden rule (see Van Damme, 2014). However, in our legal harmonization game, any Berge equilibrium is a weak golden rule.

Van Damme (2014) also considers what he calls a strong golden rule (equation 4, page 9). The definition of strong golden rule is exactly similar to the definition of a Kant equilibrium in the sense of Laffont (1975). Then, there is therefore no strong golden rule for our game.

We summarize the above discussion of the golden rule in the following proposition:

**Proposition 6.** Let \( x \) be a real number. Then \( x \) is a Berge equilibrium of the legal harmonization game if and only if it is a golden rule. Moreover, there is no strong golden rule for this game.

Golden rule do not give additional insights relatively to the Berge equilibrium.

5 Conclusion

While legal unification is observed in practice, it is difficult to rationalize it using standard economic tools. Welfare arguments can be invoked but unification is not necessarily dominated by non cooperation and can then be preferred if a first-best situation is not accessible. To solve the paradox, we have proposed a solution relying on less standard analytical tools.

Several objections can be made about the paradox of legal unification developed in this paper. First, a possible source of this paradox can lie in the standard assumption of smooth preferences. As Crettez, Deffains, and Musy (2013) have shown in a 2-country setup, with non-smooth preferences, countries can choose to choose the same legal system under some conditions. However, such preferences are not standard in economics and more work should be necessary to rationalize them in our context, especially if the implications of this assumption are crucial.

Second, the solution proposed to solve the unification paradox yields another paradox: legal harmonization is never chosen in a Berge equilibrium of our legal standardization game.

To tackle this new paradox it might be fruitful to recall that agents, but also countries, often choose situation-specific behavior rules. Following Gauthier (1986) and Courtois, Nessah, and Tazdaït (2015), we may assume that agents choose a disposition to play a game (e.g., they choose between a cooperative behavior and a less-cooperative behavior. For instance, we may assume that players choose the disposition that produces the highest expected utility given the expected disposition of the others. The investigation of the disposition approach to analyze legal standardisation games is a natural topic for future research.\(^{10}\)

References


\(^{10}\) Colman et al. (2011), page 136, notice that “if the Berge rule is appropriate to understand human behaviors in several situations, while the Nash rule may be more appropriate in others, we suspect that additional rules may complement these two.”
— (2015). “How we cooperate... perhaps”.

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A  Proof of the Propositions

Proof of Proposition 1

Proof. Let $X^*$ be a solution of problem 6. Then the following necessary condition is satisfied:

$$\beta_i \left( \frac{\partial U^i}{\partial x_i} (x_i^*, \theta_i) + \frac{\partial V^i}{\partial x_i} (X^*) \right) + \sum_{j \neq i} \beta_j \frac{\partial V^j}{\partial x_i} (X^*) = 0, \text{ for all } i. \tag{12}$$

Assume by way of contradiction that $x_j^* = x^*$ for all $j$. By the single-peak property, we have:

$$\sum_{j \neq i} \beta_j \frac{\partial V^j}{\partial x_i} (X^*) = 0, \text{ for all } i. \tag{13}$$

Therefore we obtain that:

$$\beta_i \left( \frac{\partial U^i}{\partial x_i} (x^*, \theta_i) + \frac{\partial V^i}{\partial x_i} (X^*) \right) = 0, \text{ for all } i. \tag{14}$$

From equation (2), we have $\frac{\partial V^i}{\partial x_i} (X^*) = 0$. Equation (14), thus reduces to $\beta_i \frac{\partial U^i}{\partial x_i} (x^*, \theta_i) = 0$. However, since all the $\beta_i$ are non nill and all the $\theta_i$ are different, the conditions $\frac{\partial U^i}{\partial x_i} (x^*, \theta_i)$ imply that $x^* = \theta_i$ for all $i$. This is a contradiction. \qed

Proof of Proposition 2

Proof. Assume by way of contradiction that there is a Nash equilibrium $x_i^* = x_{-i}^* = x$, when the countries’ objective functions of any player $i$ are given by equation (8). In any Nash equilibrium of the game, the following condition holds for any player $i$:

$$\sum_{j=1}^{N} \frac{\partial T^i}{\partial x_i} \frac{\partial U^j}{\partial x_i} = 0. \tag{15}$$

where $\frac{\partial U^i(X)}{\partial x_i} = \frac{\partial V^j(X)}{\partial x_i} (j \neq i)$ and $\frac{\partial U^i(X)}{\partial x_i} = \frac{\partial U^i(x_i, \theta_i)}{\partial x_i} + \frac{\partial V^i(X)}{\partial x_i}$. When legal unification is realized, $x_i = x_j = x$ and then

$$\frac{\partial V^j(X)}{\partial x_i} = \frac{\partial V^i(X)}{\partial x_i} = 0. \tag{16}$$
Therefore equation (15) reduces to
\[ \frac{\partial T^i}{\partial U^i} \frac{\partial U^i(x, \theta_i)}{\partial x_i} = 0, \] (17)

Since by assumption \( T^i \) is positive, the above equation implies that \( \frac{\partial U^i(x, \theta_i)}{\partial x_i} = 0 \) for all \( i \).

But these conditions imply in turn that \( x = \theta_i, \forall i, \forall j, i \neq j \). Since \( \theta_i \neq \theta_j \) this is a contradiction. \( \square \)

**Proof of Proposition 3**

**Proof.** Assume by way of contradiction that there exists an additive Kantian equilibrium where legal unification is realized. Then, there is a real \( x \) such that for all \( i \), the following condition is satisfied when \( \alpha = 0 \)
\[ a_i \frac{\partial U^i}{\partial x_i}(x_i, \theta_i) + a_i \frac{\partial V^i}{\partial x_i}(x_1) + \sum_{i \neq j} a_j \frac{\partial V^i}{\partial x_j}(x_1) = 0. \] (18)

Necessarily, we have
\[ a_i \frac{\partial V^i}{\partial x_i}(x_1) + \sum_{i \neq j} a_j \frac{\partial V^i}{\partial x_j}(x_1) = 0. \] (19)

Since \( a_i \neq 0 \), then \( \frac{\partial U^i}{\partial x_i}(x_i, \theta_i) = 0 \). But this implies that \( x = \theta_i \). Since there the \( \theta_i \) are all different we obtain a contradiction. \( \square \)

**Proof of Proposition 4**

**Proof.** Let \( X^b \) be a berge equilibrium. Let \( x^b_i \) be the decision assigned to country \( i \). Using the single peak property of function \( V^i \), in a Berge equilibrium we find that: \( x^b_j = x^b_i \) for all \( j \neq i \). This properties holds for all \( i \). Thus all the countries choose the same decisions. Therefore in a Berge equilibrium legal unification prevails. \( \square \)

**Proof of Proposition 5**

**Proof.** Let \( x \) be a real number. Assume that \( x_i = x \). Then since \( V^i \) satisfies the single peak property, \( U^i \) is maximized with respect to \( X_{-i} \) when \( x_k = x \) for all \( k \neq i \). Since this is true for all \( i \), we have proved that \( x_i = x \) for all \( i \) is a Berge equilibrium. \( \square \)