Dynamic Vertical Collusion

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Abstract: We show that when retailers and their supplier all care about future profits, collusion involving all of them is easier to sustain in a repeated game than collusion among retailers. This is so even when the supplier is as impatient as retailers are and even though vertical contracts are assumed to be secret. We also find that the more the retailers and the supplier care about future profits, retailers obtain a higher share of the monopoly profits. Furthermore, vertical collusion can enable the supplier to collect a higher wholesale price and make higher profits even when retailers have the bargaining power. Sustaining collusion requires retailers to commit to deal exclusively with the joint supplier and to charge slotting allowances.

Keywords: vertical relations, tacit collusion, opportunism, slotting allowances

JEL Classification Numbers: L41, L42, K21, D8

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1. Introduction

This paper asks whether and how ongoing collusion involving not only retailers, but also their joint supplier (all of whom are strategic players caring about collusive profits) is more sustainable than collusion among retailers not involving the supplier. Often, in a certain geographic area, retailers fiercely compete over end consumers. Retailers would prefer to collude at the expense of consumers, but competition among them is often too intense to support such collusion. Retailers typically buy from a joint supplier, where all firms interact repeatedly. The supplier is typically a strategic player too, who, like retailers, cares about future profits. This raises the question: can including the supplier in the collusive scheme improve the prospects of collusion?

Intuitively, when competing retailers do not place a high value on future profits, they may benefit from including a more patient supplier in their collusive scheme. It seems counterintuitive, however, that including a supplier that is as impatient as retailers are in the collusive scheme can help sustain it. After all, a short-sighted supplier, which can gain from deviating from the collusive scheme, may at first blush seem to be more of a burden to the collusive scheme than an asset.

Also, how can such a collusive scheme survive when the contracts between the supplier and each retailer is not observable to the other retailer? Had each retailer been able to observe the other retailer’s contract with the supplier, retailers could have made a credible commitment to each other to charge a high retail price, by paying the supplier an observable high wholesale price. But normally, vertical contracts between suppliers and retailers are not publicly observable, so one retailer does not know whether the supplier granted a secret discount to a competing retailer. Importantly, exchange of information among retailers competing in a downstream market regarding the terms of their contracts with a supplier is an antitrust violation.\(^1\) Since discounts given by the supplier to a retailer are secret, they encourage the

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\(^1\) See Department of Justice/Federal Trade Commission (2000) (stressing that the exchange of current or future, firm specific, information about costs is most likely to raise competitive concerns); European Commission (2011) (“the exchange of commercially sensitive information such as purchase prices and volumes … may facilitate coordination with regard to sales prices and output and thus lead to a collusive outcome on the selling markets”); Federal Trade Commission (2011) (“If the information exchanged is competitively sensitive—that is, if it is information that a company would not normally share with its competitors in a competitive marketplace, such as … supplier or cost information … or other similar information—companies should establish appropriate firewalls or other safeguards to ensure that the companies remain appropriately competitive throughout their cooperation.”); OECD (2010) (discussing an antitrust case brought by the South African Competition Commission and condemning information exchanges among competing buyers of raw milk regarding the prices paid to suppliers as a violation of the section forbidding illegal agreements); New Zealand Commerce Commission (2014) (warning that information exchanges such as “… discussing supplier interactions with a competitor create an environment in which anti-competitive agreements or conduct can easily emerge. This creates significant risk for the parties involved, including employees. Such exchanges and discussions should be avoided.”
retailer to charge a low retail price. Also, when the supplier is tempted to make such secret price cuts in favor of one retailer at the expense of the other retailer, even the supplier of a strong brand finds it difficult to commit to charging a high wholesale price.

We show that when two competing retailers and a joint supplier interact repeatedly, and all of them are strategic players who equally care about future profits, they can increase their profits (at the expense of consumers) by engaging in a collusive scheme involving retailers and the supplier as well. We refer to such a scheme as “vertical collusion”. Each of the three firms has a short-run incentive to deviate from collusion and increase its own current-period profit at the expense of the other two, yet they collude because they all gain a share of future collusive profits, should they adhere to the collusive scheme in the current period. The three firms manage to do so even when retailers are too short-sighted to maintain standard horizontal collusion between themselves and even when the supplier is as short-sighted as the retailers are. Moreover, vertical collusion holds despite the fact that vertical contracts are secret. The supplier is willing and able to participate in the vertical collusion scheme because it enables the supplier to collect higher wholesale prices and make higher profits than absent collusion. Hence, even though retailers do not observe the supplier's price cuts to one retailer at the expense of the other, the supplier himself has an incentive to police its own and retailers' adherence to the collusive scheme.

In the vertical collusive scheme we identify as an equilibrium, the higher wholesale price the supplier charges retailers reduces their short-term profits from deviating from collusion. At the same time, the supplier pays the retailers “slotting allowances” (fixed fees paid by suppliers to retailers in exchange for shelf space, promotional activities, and the like) in order to raise their long-term profits from collusion. The supplier’s short-term gain from deviating from the collusive scheme involves rejecting one of the retailers’ contracts and saving the slotting allowances owed to this retailer. But if the supplier deviates, it sacrifices the ability to charge a higher wholesale price in future periods. The parties construct the vertical contracts so as to deter all of them from deviating from the collusive scheme. When a retailer attempts to deviate from the collusive price by offering the supplier a different vertical contract, the supplier, who also benefits from maintaining the collusive scheme, rejects the retailer’s offer.

Our base model considers undifferentiated retailers. We find that for any positive discount factor, there is an equilibrium in which retailers charge the monopoly price and earn positive profits. The supplier earns his share of the vertical collusive scheme by being able to collect a higher wholesale price and making a higher profit than absent collusion. We show that the more the supplier and retailers care about the future, retailers can maintain a higher share of collusive profits. The level of slotting allowances, however, is non-monotonic in the firms' discount factor: the more firms care about future profits, the level of slotting allowances first increases and then decreases.
We then show that the existence of a competing supplier causes the collusive scheme to break down. Retailers can restore the ability to sustain vertical collusion if they can commit to buy exclusively from one of the suppliers. We study whether such exclusivity can be maintained in a collusive equilibrium.

We then extend our base model to the case of differentiated retailers. We find that our result according to which vertical collusion is easier to sustain than ordinary horizontal collusion carries over to this case as well. Unlike in the case of homogenous retailers, however, the three firms cannot sustain vertical collusion for any discount factor. We find that the less differentiated retailers are, vertical collusion becomes sustainable for a wider set of discount factors.

With regard to policy implications, our results shed a new (and negative) light on various vertical restraints. In our framework, slotting allowances, coupled by exclusive dealing by retailers with a sole supplier, facilitate the vertical collusion scheme even though vertical contracts are secret. Current literature implies that such practices can facilitate downstream collusion only when vertical contracts are observable. This implies that slotting allowances and exclusive dealing deserve stricter antitrust treatment than currently believed.  

The results also imply that the “Chicago School” approach advocating lenient treatment of vertical restraints that eliminate downstream competition may not be justified. In particular, resale prices dictated or suggested by suppliers may equal the monopoly price, to the detriment of consumers, rather than be set at a level merely stimulating efficiencies in distribution. The supplier in our model pays the retailers slotting allowances, and hence the supplier’s profits stem solely from the wholesale price it charges. It allegedly follows, according to the Chicago School’s approach, that the supplier would want retail prices to be as low as possible, to maximize the number of units sold. Our results, however, imply that the supplier may want to dictate or suggest to the retailers to charge the monopoly retail price, since this is what drives the vertical collusive scheme.

Our paper is related to several strands of the economic literature. The first strand involves vertical relations in a repeated infinite horizon game. Asker and Bar-Isaac (2014) show that when entry of a new supplier requires accommodation by retailers, an incumbent supplier can exclude the new supplier by offering retailers, on an ongoing basis, part of his monopoly profits, via vertical practices such as resale price maintenance, slotting fees, and exclusive territories. Because retailers in their model care about future profits, they may prefer to keep a new supplier out of the market, so as to continue receiving a portion of the incumbent supplier’s profits.

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2 At the same time, Chu (1992), Lariviere and Padmanabhan (1997), Desai (2000) and Yehezkel (2014) show that slotting allowances may also have the welfare enhancing effect of enabling suppliers to convey information to retailers concerning demand. See also Federal Trade Commission (2001, 2003), and European Commission (2012) discussing some of the pro’s and con’s of slotting allowances.
Another part of this literature examines collusion among retailers, where suppliers are myopic. In particular, Normann (2009) and Nocke and White (2010) find that vertical integration can facilitate downstream collusion between a vertically integrated retailer and independent retailers. Piccolo and Miklós-Thal (2012) show that retailers with bargaining power can collude by offering perfectly competitive suppliers a high wholesale price and negative fixed fees. Doyle and Han (2012) consider retailers that can sustain downstream collusion by forming a buyer group that jointly offers contracts to suppliers. The rest of this literature studies collusion among suppliers, where retailers are myopic: Jullien and Rey (2007) consider an infinite horizon model with competing suppliers where each supplier sells to a different retailer and offers it a secret contract. Their paper studies how suppliers can use resale price maintenance to facilitate collusion among the suppliers, in the presence of stochastic demand shocks. Nocke and White (2007) consider collusion among upstream firms and the effect vertical integration has on such collusion. Reisinger and Thomes (2015) analyze a repeated game between two competing and long-lived manufacturers that have secret contracts with myopic retailers. They find that colluding through independent, competing retailers is easier to sustain and more profitable to the manufacturers than colluding through a joint retailer. Schinkel, Tuinstra and Rüggeberg (2007) consider collusion among suppliers in which suppliers can forward some of the collusive profits to downstream firms in order to avoid private damages claims. Piccolo and Reisinger (2011) find that exclusive territories agreements between suppliers and retailers can facilitate collusion among suppliers. The main difference between our paper and this literature is that we examine collusion involving the whole vertical chain: supplier and retailers alike, who are all forward looking, and all have a short run incentive to deviate from collusion which is balanced against a long – run incentive to maintain the collusive equilibrium.

The second strand of the literature concerns static games in which vertical contracts serve as a devise for reducing price competition between retailers. Bonanno and Vickers (1988) consider vertical contracts when suppliers have the bargaining power. They find that suppliers use two-part tariffs that include a wholesale price above marginal cost in order to relax downstream competition, and a positive fixed fee, to collect the retailers’ profits. Shaffer (1991) and (2005), Innes and Hamilton (2006), Rey, Miklós-Thal and Vergé (2011) and Rey and Whinston (2012) consider the case where retailers have buyer power. In such a case, retailers pay wholesale prices above marginal cost in order to relax downstream competition and suppliers pay fixed fees to retailers.

The difference between our paper and this strand of the literature is that we study a repeated game rather than a static game. This enables us to introduce the concept of vertical collusion, where the supplier, as well as retailers, care about future profits. Also, in this literature, vertical contracts are observable to retailers. We consider the prevalent case where vertical contracts are unobservable.
The third strand of literature involves static vertical relations in which a supplier behaves opportunistically by granting price concessions to one retailer at the expense of the other. Hart and Tirole (1990), O’Brien and Shaffer (1992), McAfee and Schwartz (1994) and Rey and Vergé (2004) consider suppliers that make secret contract offers to retailers. They find that a supplier may behave opportunistically (depending on the retailers’ beliefs regarding the supplier’s offer to the competing retailers) and offer secret discounts to retailers. Anticipating this, retailers do not agree to pay high wholesale prices and the supplier cannot implement the monopoly outcome. The vertical collusive scheme we identify resolves the opportunism problem exposed in the above literature and restores the supplier’s power to charge high wholesale prices. If a supplier and one of the retailers in our model behave opportunistically in a certain period, vertical collusion breaks down in the next periods. Since the two retailers and the supplier all care about future profits, this serves as a punishment against opportunistic behavior.

2. The model
Consider two downstream retailers, \( R_1 \) and \( R_2 \) that compete in prices. In our base model, we focus on the extreme case where retailers are homogeneous. Doing so enables us to deliver our main results in a clear and tractable manner. In section 5, we show that the qualitative results of the base model extend to markets with differentiated retailers.

Retailers can obtain a homogeneous product from an upstream supplier. Production and retail costs are zero. Consumers’ demand for the product is \( Q(p) \), where \( p \) is the final price and \( pQ(p) \) is concave in \( p \). Let \( p^* \) and \( Q^* \) denote the monopoly price and quantity, where \( p^* \) maximizes \( pQ(p) \) and \( Q^* = Q(p^*) \). The monopoly profit is \( p^*Q^* \).

The two retailers and the supplier interact for an infinite number of periods and have a discount factor, \( \delta \), where \( 0 \leq \delta \leq 1 \). The timing of each period is as follows:

- **Stage 1:** Retailers offer a take-it-or-leave-it contract to the supplier (simultaneously and non-cooperatively). Each \( R_i \) offers a contract \( (w_i, T_i) \), where \( w_i \) is the wholesale price and \( T_i \) is a fixed payment from \( R_i \) to the supplier that can be positive or negative. In the latter case the supplier pays slotting allowances to \( R_i \). The supplier observes the offers and decides whether to accept one, both or none. All of the features of the bilateral contracting between \( R_i \) and the supplier are unobservable to \( R_j (j \neq i) \) throughout the game. Moreover, \( R_i \) cannot know whether \( R_j \) signed a contract with the supplier until the end of the period, when retail prices are observable. The contract offer is valid for the current period only.\(^3\)

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\(^3\) See Piercy (2009), claiming that large supermarket chains in the UK often change contractual terms, including the wholesale price and slotting allowances, on a regular basis, e.g., via e-mail correspondence; Lindgreen, Hingley and Vanhamme (2009), discussing evidence from suppliers regarding large
• **Stage 2:** The two retailers set their retail prices for the current period, $p_1$ and $p_2$, simultaneously and non-cooperatively. Consumers buy from the cheapest retailer. In case $p_1 = p_2$, each retailer gains half of the demand. At the end of the stage, retail prices become common knowledge (but again retailers cannot observe the contract offers). If in stage 1 the supplier and $R_i$ didn’t sign a contract, $R_i$ only learns about it at the end of the period, when $R_i$ observes that $R_i$ didn’t set a retail price for the supplier’s product (or equivalently charged $p_j = \infty$). Still, $R_i$ cannot know why $R_j$ and the supplier didn’t sign a contract (that is, $R_i$ doesn’t know whether the supplier, $R_j$, or both, deviated from the equilibrium strategy).

We consider pure-strategy, perfect Bayesian-Nash equilibria. We focus on symmetric equilibria, in which along the equilibrium path both retailers choose the same strategy, equally share the market and earn identical profits. We allow an individual retailer to deviate unilaterally outside the equilibrium path.

When there is no upstream supplier and the product is available to retailers at marginal costs, retailers only play the second stage in every period, in which they decide on retail prices, and therefore the game becomes a standard infinitely-repeated Bertrand game with two identical firms. Then, a standard result is that horizontal collusion over the monopoly price is possible if:

$$\frac{p^*Q^*\frac{1}{2}}{1-\delta} > p^*Q^* \iff \delta > \frac{1}{2},$$

where the left hand side is the retailer’s sum of infinite discounted profit from colluding on the monopoly price and gaining half of the demand and the right hand side is the retailer’s profit from slightly undercutting the monopoly price and gaining all the demand in the current period, followed by a perfectly competitive Bertrand game with zero profits in all future periods. Given this benchmark value of $\delta = 1/2$, we ask whether the prospects of vertical collusion, involving retailers and the supplier as well, are higher than horizontal collusion between the retailers. This analysis will take account of the fact that one retailer’s two-part-tariffs are unobservable to the competing retailer throughout the game and both retailers and the supplier equally care about future profits.

#### 3. Competitive static equilibrium benchmark

The result that retailers cannot earn positive profits in any competitive equilibrium suggests that in a dynamic, infinitely repeated game, retailers have an incentive to engage in tacit collusion. They cannot sustain horizontal collusion, however, for discounts factors below half. The supplier has an incentive to participate in a collusive equilibrium when the supplier expects that otherwise retailers will play a competitive equilibrium involving $\pi^c < p^*Q^*$. In this section we solve for a competitive equilibrium benchmark in which the three firms have $\delta = 0$. This can also be an equilibrium when $\delta > 0$ and the three firms expect that their strategies in the current period will not affect the future. This benchmark is needed for our analysis because we will assume that an observable deviation from vertical collusion will result in playing the competitive equilibrium in all future periods. The main result of this section is that in the static game price competition dissipates all of the retailers’ profits. Moreover, since contracts are secret and the supplier has an incentive to act opportunistically, there are equilibria in which the supplier earns below the monopoly profits.

In a symmetric equilibrium, in stage 1 both retailers offer the contract $(T^c, w^c)$ that the supplier accepts. Then, in stage 2, both retailers set $p^c$ and equally split the market. Each retailer earns $(p^c - w^c)Q(p^c)/2 - T^c$ and the supplier earns $w^cQ(p^c) + 2T^c$. Since vertical contracts are secret, there are multiple equilibria, depending on firms’ beliefs regarding off-equilibrium strategies. In what follows, we characterize the qualitative features of these equilibria.

First, notice that in any such equilibrium $p^c = w^c$, because in the second stage retailers play the Bertrand equilibrium given $w^c$. Therefore, there is no competitive equilibrium with $T^c > 0$, because retailers will not agree to pay a positive fixed fee in stage 1, given that they don’t expect to earn positive profits in stage 2. There is also no competitive equilibrium with $T^c < 0$. To see why, notice that the supplier can profitably deviate from such an equilibrium by accepting only one of the contracts, say, the contract of $R_i$. $R_i$ expects that in equilibrium both of the retailers’ offers are accepted by the supplier. $R_i$ cannot observe the supplier’s deviation of not accepting $R_j$’s contract. Accordingly, in stage 2 $R_i$ sets the equilibrium price $p^c$. The supplier’s profit is $w^cQ(w^c) + T^c$ -- higher than the profit from accepting both offers, $w^cQ(w^c) + 2T^c$ whenever $T^c < 0$. Therefore, in all competitive equilibria, $T^c = 0$.

Next, consider the equilibrium wholesale price in the competitive static equilibrium benchmark, $w^c$. The equilibrium value of $w^c$ depends on the beliefs regarding out-of-equilibrium strategies. In particular, when $R_i$ makes a deviating offer that the supplier accepts, $R_i$ cannot observe whether the supplier accepted $R_j$’s equilibrium offer, and therefore needs to form beliefs concerning the supplier’s response to such a deviation. Suppose that the three firms share the following belief: When $R_i$’s offer to the supplier deviates from the equilibrium contract, making it worthwhile for the supplier to reject $R_i$’s offer, the supplier indeed rejects $R_i$’s offer. These beliefs are close in nature to the “wary beliefs” discussed in McAfee and
Schwartz (1994) and in what follows we adopt the same terminology. At first blush, it might be thought that the optimal deviation for Ri and the supplier is to a contract with wi = 0, that the supplier accepts, while rejecting Rj’s offer. With such a deviation, Ri can set the monopoly price p*, maximize the joint profits of the supplier and himself, and share these profits with the supplier via Ti. Given that Ri offered to pay the supplier wi = 0, however, the supplier has the incentive to behave opportunistically and accept Rj’s offer to pay a positive wholesale price. Under “wary beliefs”, Ri expects such opportunistic behavior, and hence will not offer to pay the supplier wi = 0.

The following lemma characterizes the set of competitive static equilibria under wary beliefs. It shows that in the competitive benchmark case, retailers make zero profits, while the supplier makes a positive profit:

**Lemma 1:** Suppose that \( \delta = 0 \). Then, under wary beliefs, there are multiple equilibria with the contracts \((T^C, w^C) = (0, w^C), w^C \in [w_L, p^*] \), where \( w_L \) is the lowest solution to

\[
\max_{w_i} \{w_i Q(p(w_i))\} < w^C Q(w^C) \quad \text{where} \quad p(w_i) = \arg \max_p \{(p - w_i)Q(p)\},
\]

and \( 0 < w_L \leq p^* \). In equilibrium, retailers set \( p^C \) and earn 0 and the supplier earns \( \pi^C \equiv w^C Q(w^C) \), \( \pi^C \in [w_L Q(p(w_L)), p^* Q^*] \).

**Proof:** see the Appendix.

Notice that there is a competitive static equilibrium in which the supplier earns the monopoly profit, \( p^* Q^* \). Intuitively, this equilibrium holds because the supplier can implement the monopoly outcome by dealing with only one of the retailers. Given that Rj offers \( w_j = p^* \) and expects that Ri does the same, Ri cannot profitably deviate to any other contract than \( w_i = p^* \), because the supplier can earn \( p^* Q^* \) by accepting Rj’s contract offer alone. In what follows, we rule out the equilibrium with \( \pi^C = p^* Q^* \) for two reasons. First, this equilibrium is an artifact of our simplifying assumption that retailers are homogeneous. The equilibrium does not hold when retailers are even slightly differentiated, because in such a case the supplier needs to deal with both retailers in order to implement the monopoly outcome. The second reason is that as shown in the next section, retailers’ profits in the collusive equilibrium are decreasing with \( \pi^C \). Consequently, retailers have an incentive to coordinate on a punishment strategy in which following a deviation from collusion they play the competitive equilibrium that provides the supplier with the lowest profit possible.

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4 In McAfee and Schwartz (1994), under "wary beliefs" a retailer believes that if the supplier offered him a contract that deviates from the equilibrium contract, the supplier offers the competing retailer a contract that maximizes the joint profit of the supplier and competing retailer.
As we will show in the next section, other than our assumption that \( \pi^C < p^*Q^* \), the qualitative features of the collusive equilibrium do not depend on the value of \( \pi^C \).

4. Vertical collusive equilibrium with infinitely repeated interaction

4.1. The condition for sustainability of the collusive equilibrium

The result that retailers cannot earn positive profits in any competitive equilibrium suggests that in a dynamic, infinitely repeated game, retailers have an incentive to engage in tacit collusion. They cannot sustain horizontal collusion, however, for discount factors below half. The supplier, for its part, has an incentive to participate in a collusive equilibrium when it expects that otherwise retailers will play a competitive equilibrium involving \( \pi^C < p^*Q^* \). In this section, we solve for the collusive equilibrium in an infinitely repeated game when \( 1 \geq \delta > 0 \). In this equilibrium, in the first stage both retailers offer the same equilibrium contract, \((w^*,T^*)\) that the supplier accepts. Then, in stage 2, both retailers set the monopoly price, \( p^* \), and equally split the monopoly quantity, \( Q^* \). Given an equilibrium \( w^* \), each retailer earns in every period \( \pi_R(w^*) = (p^* - w^*)Q^*/2 - T^* \) and the supplier earns in every period \( \pi_S(w^*) = w^*Q^* + 2T^* \).

In order to support the collusive scheme, the contract \((w^*,T^*)\) must prevent deviations from this scheme. \( R_i \) can observe whether \( R_j \) deviated from the monopoly retail price \( p^* \), thereby dominating the downstream market. \( R_i \) cannot observe, however, whether this deviation is a result of \( R_j \) offering the supplier a different contract than \((w^*,T^*)\), which motivates \( R_j \) to deviate from the monopoly price, or whether \( R_j \) offered the supplier the equilibrium contract \((w^*,T^*)\), but nevertheless undercut the monopoly price. It is only the supplier and \( R_j \) that will know which type of deviation occurred. \( R_i \) can also observe whether \( R_j \) did not carry the product in a certain period. \( R_i \) cannot tell, however, whether this is a result of a deviation by \( R_j \) (i.e., \( R_j \) offered a different contract than \((w^*,T^*)\) that the supplier rejected) or by the supplier (i.e., \( R_j \) offered the equilibrium contract \((w^*,T^*)\), but the supplier rejected). Finally, another type of deviation is when \( R_i \) offers a contract different than \((w^*,T^*)\) that the supplier accepted, but then \( R_i \) continued to set \( p^* \). \( R_j \) will never learn of this deviation, since contracts are secret. Because of the dynamic nature of the game and the asymmetry in information, there are multiple collusive equilibria. We therefore make the following restrictions. First, suppose that whenever a publicly observable deviation occurs (i.e., a retailer sets a different price than \( p^* \) or does not carry the product), retailers play the competitive equilibrium defined in section 3 in all future

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5 It is possible to show that if retailers have "passive beliefs" according to the definition in McAfee and Schwartz (1994), then any \( w^c \in [0, p^*] \) and therefore any \( \pi^c \in [0, p^*Q^*] \) can be an equilibrium.
periods. Second, since we concentrate here on retailers with strong bargaining power, we focus on outcomes that provide retailers with the highest share of the monopoly profit that ensures the supplier at least its competitive equilibrium profit, \( \pi^C \).

To solve for the collusive equilibrium, we first consider necessary conditions on \((w^*, T^*)\). Then, we show that these conditions are also sufficient. The first condition is that once retailers offered a contract \((w^*, T^*)\) that the supplier accepted, \( R_i \) indeed plays in stage 2 the monopoly price \( p^* \) rather than deviating to a slightly lower price. By deviating, \( R_i \) gains all the demand in the current period, but stops future collusion. \( R_i \) will not deviate from collusion in the second stage if:

\[
(p^* - w^*) \frac{1}{2} Q^* + \frac{\delta}{1 - \delta} \left( (p^* - w^*) \frac{1}{2} Q^* - T^* \right) \geq (p^* - w^*) Q^*,
\]

where the left hand side is \( R_i \)’s profit from maintaining collusion and the right hand side is \( R_i \)’s profit from deviating. Notice that condition (2) is affected only by the retailers’ discount factor and not by the supplier’s, because this constraint involves a deviation by a retailer assuming the supplier had not deviated: he played the equilibrium strategy and accepted the two equilibrium contract offers in stage 1.

The second necessary condition is the supplier’s participation constraint:

\[
\frac{w^* Q^* + 2T^*}{1 - \delta} = \frac{w^* Q^* + T^*}{1 - \delta} \pi^C.
\]

The left hand side is the supplier’s profit from accepting the two equilibrium contracts and thereby maintaining collusion. The right hand side is the supplier’s profit from accepting only one of the contracts. If the supplier rejects \( R_i \)’s offer, \( R_j \) can detect this deviation only at the end of stage 2, when \( R_j \) observes that \( R_i \) doesn’t offer the product. Therefore, in stage 2 \( R_j \) will still charge the monopoly price \( p^* \) and sell \( Q^* \), implying that the supplier earns in the current period \( w^* Q^* + T^* \) and collusion breaks down in all future periods, in which the supplier earns \( \pi^C \). If the left hand side of (3) is higher than the right hand side, then \( R_i \) has the incentive to deviate to a contract with a lower \( T_i \), that the supplier would accept, since even with this lower \( T_i \), the supplier prefers collusion to deviation. If the right hand side of (3) is higher than the left hand side, then when both retailers offer the equilibrium contract, the supplier will deviate from the equilibrium strategy in stage 2 and accept only one of the contracts. Therefore, condition (3)

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\(^{6}\) We consider an alternative trigger strategy in section 4.5.

\(^{7}\) Retailers may also be able to coordinate on the competitive equilibrium outcome and choose the lowest \( \pi^C \) possible, \( w_l Q(w_l) \). Our qualitative results do not rely on the size of \( \pi^C \), however, as long as collusion is weakly beneficial to all three firms (i.e., \( \pi^C < p^* Q^* \)). Accordingly, we solve for the collusive equilibrium for any arbitrary \( \pi^C \).
must hold in equality. Notice that this condition is affected by the supplier's discount factor only, and not by the retailers' discount factor, because it deals with the supplier's deviation given that retailers had offered the equilibrium contracts.

Extracting $T^*$ from (3) and substituting into $\pi_S(w^*)$, we can rewrite the supplier and retailers’ one-period profits as a function of $w^*$ as:

$$\pi_R(w^*) = \left( p^* - \frac{1-\delta}{1+\delta} w^* \right) Q^*/2 - \frac{\delta}{1+\delta} \pi^C, \quad \pi_S(w^*) = \frac{1-\delta}{1+\delta} w^* Q^* + \frac{2\delta}{1+\delta} \pi^C. \quad (4)$$

As can be expected, $\pi_R(w^*)$ is decreasing in $w^*$ while $\pi_S(w^*)$ is increasing in $w^*$.

The two conditions above ensure that the supplier accepts the two equilibrium contracts and that each retailer sets $p^*$ if the supplier accepts its equilibrium contract. The remaining requirement is that $R_i$ does not find it profitable to deviate in stage 1 to any other contract. To examine this requirement, suppose that in a certain period, $R_i$ offers some deviating contract, $(w_i, T_i) \neq (w^*, T^*)$. As in the competitive, static case, the benefit to $R_i$ and the supplier from such a deviation depend on their out-of-equilibrium beliefs concerning each other's future strategies given the deviation. In particular, when deciding whether to accept the deviating contract, the supplier needs to form beliefs on whether this contract will motivate $R_i$ to set the monopoly retail price or undercut it. Likewise, if the supplier accepts the deviating contract, $R_i$ needs to form beliefs on whether the supplier accepts the equilibrium contract of $R_j$ as well. We apply wary beliefs as follows. Suppose $R_i$ deviated to $(w_i, T_i) \neq (w^*, T^*)$. If this deviation induces $R_i$ to undercut the monopoly retail price, the supplier correctly anticipates this fact and rejects $R_j$’s contract (since $R_j$ will make no sales). $R_j$, in turn, understands that the supplier will behave in this manner. Conversely, if the deviation does not induce $R_i$ to undercut the monopoly retail price, the supplier anticipates this and accepts $R_j$’s contract, and $R_i$ again understands that the supplier will behave in this manner.\footnote{It is possible to show that when the supplier is "naive" and wrongly anticipates that any contract deviation will not motivate $R_i$ to undercut the monopoly retail price, all collusive equilibria fail for $\delta < 1/2$.}

Proposition 1 below shows that given wary beliefs, conditions (2), (3) and $\pi_S(w^*) \geq \pi^C$, $R_i$ cannot profitably deviate to any $(w_i, T_i) \neq (w^*, T^*)$. Therefore, conditions (2), (3) and $\pi_S(w^*) \geq \pi^C$ are also sufficient for sustainability of the collusive equilibrium. Proposition 1 also characterizes the unique collusive contract that maximizes the retailers' profits subject to (2), (3) and $\pi_S(w^*) \geq \pi^C$. 

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8 It is possible to show that when the supplier is "naive" and wrongly anticipates that any contract deviation will not motivate $R_i$ to undercut the monopoly retail price, all collusive equilibria fail for $\delta < 1/2$. 

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Proposition 1: Suppose that \( \delta > 0 \). Then, under wary beliefs, there is a unique collusive equilibrium that maximizes the retailers’ profits subject to (2), (3) and \( \pi_S(w^*) > \pi_C \). In this equilibrium:

\[
\begin{align*}
w^* &= \begin{cases} 
\frac{p^* - 2\delta^2(1 - \delta)Q^*}{(1 - \delta)Q^*} ; & \delta \in (0, \frac{1}{2}] ; \\
P_C/Q^* ; & \delta \in [\frac{1}{2}, 1] ;
\end{cases} \\
\text{and: } T^* &= \begin{cases} 
\frac{\delta}{1 - \delta}(1 - 2\delta)(1 - \delta)Q^* - \pi_C) ; & \delta \in (0, \frac{1}{2}] ; \\
0 ; & \delta \in [\frac{1}{2}, 1].
\end{cases}
\end{align*}
\]

Proof: see the Appendix.

Substituting (5) into (4) yields that the retailers and the supplier earn in equilibrium \( \pi_R^* = \pi_R(w^*) \) and \( \pi_S^* = \pi_S(w^*) \) where:

\[
\begin{align*}
\pi_R^* &= \begin{cases} 
\delta(p^* Q^* - \pi_C) ; & \delta \in (0, \frac{1}{2}] ; \\
\frac{1}{2}(p^* Q^* - \pi_C) ; & \delta \in [\frac{1}{2}, 1] ;
\end{cases} \\
\pi_S^* &= \begin{cases} 
(1 - 2\delta)p^* Q^* + 2\delta\pi_C ; & \delta \in (0, \frac{1}{2}] ; \\
\pi_C ; & \delta \in [\frac{1}{2}, 1].
\end{cases}
\end{align*}
\]

4.2. The features of the retailers’ most profitable collusive equilibrium

Let \( SA^* = -T^* \) denote the equilibrium slotting allowance. The following corollary describes the features of the retailers’ most profitable vertical collusion equilibrium, while figure 1 illustrates the retailers' most profitable vertical collusion equilibrium as a function of \( \delta \).

Corollary 1: In the retailers’ most profitable collusive equilibrium:

(i) For \( \delta \in (0, 1/2] \):

- retailers’ one-period profits are increasing with \( \delta \) while the supplier’s one-period profit is decreasing with \( \delta \);
- the equilibrium wholesale price is decreasing with \( \delta \);
- The supplier pays retailers slotting allowances: \( SA^* > 0 \). The slotting allowances are an inverse U-shape function of \( \delta \).

(ii) For \( \delta \in [1/2, 1] \):

- the equilibrium wholesale price and the firms’ profits are independent of \( \delta \) and retailers do not charge slotting allowances: \( T^* = 0 \);
- the supplier earns its reservation profit (from the competitive equilibrium) and retailers earn the remaining monopoly profits.
Figure 1: The features of the retailers’ most profitable equilibrium as a function of $\delta$
Figure 1 and part (i) of Corollary 1 reveal that at $\delta \to 0$, $w^* \to p^*$, $SA^* \to 0$ the supplier earns most of the monopoly profits. As $\delta$ increases, $w^*$ decreases and retailers gain a higher proportion of the monopoly profits. Moreover, the equilibrium slotting allowances are an inverse U-shaped function of $\delta$. The intuition for these results is as follows. Consider first the case where $\delta = 0$. Since retailers do not care about the future, the only possible $w^*$ that motivates a retailer to set the monopoly price in stage 2 is $w^* = p^*$. For any other $w^* < p^*$, an individual retailer will deviate in stage 2 to a price slightly below $p^*$ and monopolize the market, ignoring the negative effect of doing so on future profits. Since the supplier also does not care about the future, and since $w^* = p^*$, retailers cannot charge slotting allowances. To see why, notice that if $R_i$ asks for a slotting allowance, the supplier can reject $R_i$'s contract and earn $\pi_S(w^*) = w^*Q^*$ from accepting the contract of $R_j$ and ignoring the negative effect of breaking collusion in the future. As a result, with $w^* = p^*$ and without slotting allowances, a collusive equilibrium requires the supplier to gain all of the monopoly profits. However, in such a case retailers have week incentives to participate in the collusive equilibrium to begin with.

Suppose now that $\delta$ increases slightly above 0. In this case retailers have two complementary ways to collect a positive share of the monopoly profit from the supplier. First, now $R_i$ can charge slotting allowances. If the supplier rejects $R_i$'s contract and accepts only $R_j$'s contract, the supplier earns a one-period profit close to the monopoly profit in the current period, but collusion breaks in future periods. Since now the supplier cares about the future, $R_i$ can ask for slotting allowances, which the supplier accepts, just in order to maintain collusion in the following periods.

The second option that $R_i$ can use in order to gain a positive share of the monopoly profit is by reducing $w^*$ below $p^*$. Now that retailers care about the future, they can sustain collusion even for a smaller $w^*$. Intuitively, the higher is $w^*$, the lower is $R_i$'s profit margin, and the lower is its short-term profit from deviating from $p^*$ in stage 2. To see why, notice that whenever $R_i$ sets $p^*$, $R_i$ earns in the current period a profit margin of $p^* - w^*$ on half of the monopoly quantity, while by deviating to a slightly lower price than $p^*$, $R_i$ can earn a profit margin $p^* - w^*$ on all the monopoly quantity. Accordingly, when $\delta = 0$ the only possible collusion-supporting wholesale price is $w^* = p^*$. However, when $\delta$ is slightly higher than 0, $R_i$ cares about the future, and in stage 2 will charge the monopoly price even when $w^* < p^*$. Hence, when $\delta > 0$, $R_i$ can exploit the supplier's concern about future profits (through condition (3)) in order to charge slotting allowances and can exploit its own concern about future profits (through condition (2)) in order to reduce $w^*$. Therefore, in equilibrium, retailers ask for slotting allowances and set $w^* < p^*$, both enabling them to gain a positive share of the monopoly profit. As $\delta$ increases, the supplier's incentive to maintain collusion increases, and retailers can take
advantage of it by offering a contract that allocates to them a higher share of the monopoly profit. As a result, the retailers’ profits increase with $\delta$ while the supplier’s profit decreases with $\delta$. This also explains why the equilibrium $w^*$ decreases with $\delta$. As $\delta$ increases, retailers have more of an incentive to maintain the collusive equilibrium, and therefore a lower $w^*$ is sufficient for motivating retailers not to undercut the monopoly price in stage 2.

The effect of $\delta$ on the level of slotting allowances is non-monotonic, because $\delta$ has two opposite effects on the level of slotting allowances. First, there is a positive direct effect, because the more the supplier cares about the future, the higher the slotting allowances the supplier is willing to pay to maintain collusion. Second, an indirect negative effect, because as $\delta$ increases, $w^*$ decreases. This in turn reduces the supplier’s willingness to pay slotting allowances. The first effect dominates for low values of $\delta$ while the second effect dominates for high values of $\delta$.

Part (ii) of Corollary 1 reveals that when $\delta > 1/2$, retailers sufficiently care about the future to maintain horizontal collusion, without the supplier’s participation. Accordingly, retailers keep the supplier on its profit when collusion breaks down, $\pi^C$, and earn the remaining monopoly profits. As a result, the firms’ profits and the equilibrium contract are not a function of $\delta$. The intuition follows from the benchmark case in section 2, where two firms that compete in prices can maintain horizontal collusion on their own for $\delta > 1/2$.

Corollary 1 shows that as $\delta$ increases, retailers gain a higher share of collusive profits and the supplier’s share diminishes, while when $\delta$ is small, retailers have a smaller share of the collusive profits, and most of the monopoly profits go to the supplier. This implies that even though retailers have all of the bargaining power and are asking (and receiving) slotting allowances, they are not always the main beneficiaries of the collusive scheme.

Finally, we are interested in asking whether retailers can maintain a collusive equilibrium when they cannot charge $T^* \leq 0$. Notice that the answer to this question does not directly follow from proposition 1, because this proposition only shows that the retailers’ most profitable collusive equilibrium involves slotting allowances. It is yet to be determined whether a collusive equilibrium is still possible when retailers cannot charge negative fees. In the context of this model, the following corollary shows that for $\delta \in (0, 1/2]$, firms cannot maintain any collusive equilibrium without using slotting allowances.

**Corollary 2:** If $\delta < 1/2$, then there are no contracts $(w^*, T^*)$ that can maintain a collusive equilibrium with $T^* \geq 0$.

**Proof:** see the Appendix.
The intuition for corollary 2 is that a high wholesale price has two conflicting effects on the retailers' incentive to participate in vertical collusion. First, a positive short-run effect, in that a high wholesale price decreases the profit a retailer can earn in the current period by undercutting the collusive price. Second, a long-run negative effect, because a high wholesale price (also to be paid in future periods) decreases the retailer's future profits from maintaining collusion. In order to offset the second, negative effect, future contracts need to involve fees paid by the supplier to the retailer. Notice that in the current period, the retailer earns the slotting allowance regardless of whether the retailer charges the collusive price or not. These fees are set in the first stage of the current period and will have already been paid in the second stage of the current period, when the retailer sets its price. Nevertheless, the retailer knows that if it does not charge the monopoly price in the current period, he will not receive slotting allowances in future periods. This is what induces the retailer to maintain collusion in the current period.

4.3 Competition among suppliers

Until now, we have assumed that the supplier is a monopoly. Because the monopolistic supplier cares about future profits, he enables vertical collusion even for $\delta < \frac{1}{2}$, where ordinary horizontal collusion breaks down. An important question is whether competition among suppliers causes the collusive scheme to break down. The main conclusion of this section is that retailers cannot maintain the collusive equilibrium when they have the option to buy the input from a competitive supplier.

Suppose now that the market includes a dominant supplier, $S_1$, and a competitive supply market, which consists of one or more identical suppliers, $S_2 \ldots S_n$. The dominant supplier discounts future profits by $\delta$ while the competitive suppliers are myopic. We ask whether the two retailers can sustain a collusive equilibrium in which they offer only the dominant supplier a contract ($w^*, T^*$) that the dominant supplier accepts, and then charge consumers $p^*$. As before, we assume that any observable deviation in period $t$ triggers the competitive equilibrium from period $t + 1$ onwards. We further assume that in this competitive equilibrium, all firms earn zero. That is, $\pi^C = 0$.

In order to maintain a collusive equilibrium, the collusive contract has to satisfy conditions (2) and (3). In addition, the collusive contract needs to eliminate the incentive of $R_i$ to deviate from collusion by offering the collusive contract to the dominant supplier and at the same time making a secret contract offer to a competing supplier with $w_i = T_i = 0$ (we are assuming, for now, that $R_i$ may not offer the dominant supplier to buy exclusively from it, and later relax this assumption). To see the profitability of such a unilateral deviation, suppose that $R_j$ plays according to the proposed equilibrium by offering $(w^*, T^*)$ to the dominant supplier
only, but the deviating retailer, \( R_i \), offers \((w^*, T^*)\) to the dominant supplier and at the same time makes a secret offer to \( S_2 \) with \( w_i = T_i = 0 \). The dominant supplier will accept both offers, because it is unaware of \( R_i \)'s secret offer to \( S_2 \). Hence \( R_i \) will earn a slotting allowance, \(-T^* > 0\), from the dominant supplier.

Moreover, \( R_i \) can then charge consumers a price slightly below \( p^* \), dominate the market and earn \( p^*Q^* - T^* \). If this deviation is profitable for \( R_i \) even though it breaks down collusion in all future periods, the collusive equilibrium fails. Therefore, the equilibrium requires that \( R_i \)'s discounted future profits from the collusive equilibrium are higher than a one-period deviation in which \( R_i \) buys from the competitive supplier. That is:

\[
\frac{(p^*-w)Q^*/2 - T^*}{1-\delta} \geq p^*Q^*-T^*.
\]

To see whether this condition holds, recall from equation (7) that the highest sum of discounted profits that a retailer can earn in a collusive equilibrium – the retailers' most profitable collusive equilibrium – is \( \delta p^*Q^*/(1 - \delta) \) (note that when collusion breaks down \( \pi^C = 0 \)). According to corollary 2, in any collusive equilibrium \( T^* < 0 \), and hence the lowest profit that a retailer can make by making a secret offer to a competitive supplier, is \( p^*Q^* \). However:

\[
p^*Q^* \geq \frac{\delta p^*Q^*}{1-\delta} \iff \delta \leq \frac{1}{2},
\]

implying that \( R_i \) will deviate from this collusive equilibrium by making the secret offer to the competing supplier. The following corollary summarizes this result:

**Corollary 3:** Suppose that the upstream market includes a dominant supplier and a competitive supply market. Then, if \( \delta < \frac{1}{2} \), there is no collusive equilibrium in which the two retailers sign in every period a contract with a dominant supplier.

Intuitively, for \( \delta < \frac{1}{2} \), retailers are too short-sighted and have a strong incentive to deviate from collusion. A dominant supplier is therefore needed in order to engage in vertical collusion, since otherwise collusion breaks down. But when retailers can buy the product at \( w = 0 \) from a competitive supplier, the dominant supplier’s ability to support vertical collusion is eliminated.

The result of corollary 3 continues to hold even when the competing suppliers are not myopic but rather forward looking, i.e., they have a discount factor \( \delta > 0 \). In an equilibrium in which the two retailers deal with the dominant supplier only, all other suppliers earn 0 and they do not have any outside option. Suppose \( R_i \) plays according to the proposed equilibrium by offering \((w^*, T^*)\) to the dominant supplier only, but the deviating retailer, \( R_i \), offers \((w^*, T^*)\) to the dominant supplier and at the same time makes a secret offer to \( S_2 \) with \( w_i = T_i = 0 \). Even a
forward looking $S_2$ would accept such a deviating offer, because $S_2$ earns 0 anyway if it rejects $R_i$'s offer. As before, $R_i$ could then earn $p^*Q^* - T^*$ from deviating, which is higher than $R_i$'s expected profit from the collusive equilibrium if $\delta < 1/2$.

One might expect that retailers would want to mimic the monopoly upstream market result and restore vertical collusion by signing exclusive dealing agreements with one supplier and granting him monopoly power. Such a strategy, however, is similarly vulnerable to retailers’ incentive to deviate. Suppose now that a retailer can offer a supplier an exclusive dealing contract in which the retailer commits not to buy during the relevant period from any competing supplier. The results of Corollary 3 still hold and collusion breaks down in the presence of competing suppliers. To see why, suppose that retailers can commit to an exclusive dealing contract, which is valid for one period at a time. Consider a collusive equilibrium in which in every period, each retailer offers the dominant supplier the collusive contract along with an exclusive dealing clause. The exclusive dealing obligation excludes the possibility that a retailer make an offer to the dominant supplier while ending up buying the product from a competing supplier. Nevertheless, a retailer can choose not to make an offer to the dominant supplier at all, and instead make an offer only to a competing supplier, with $w = T = 0$. The retailer's profit from deviation would no longer be $p^*Q^* - T^*$ but only $p^*Q^*$. However, it follows from (8) that the result of corollary 3 remains intact: collusion is still impossible for $\delta < 1/2$.

Hence, collusion and exclusive dealing are not practices that can sustain each other: when collusion is vulnerable to deviation by a retailer, so is exclusive dealing with a mutual supplier. There must be an external commitment mechanism that ties both retailers to the same supplier. This external commitment mechanism can take the form of a long-term contract both retailers sign with the same supplier. For example, suppliers of dominant brands are often made “category captain” of the relevant category within the branches of all leading retail chains and can use this position to eliminate rivals who may cut wholesale prices. Alternatively, one of the suppliers may have some inherent advantage. For example, he may offer a brand or product that is a “must have” brand for retailers. When retailers must purchase a portion of their requirements from a particular supplier, the supplier can relatively cheaply use loyalty rebates to induce retailers to operate exclusively with him. Another external commitment device that can secure exclusivity, despite retailers’ inherent incentive to breach it, may be a relationship-specific investment made by the retailer that ties the retailer to a particular supplier (such as

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9 We discuss the robustness of our results to the possibility of sequential offers in the conclusion.
10 See, e.g., the cases of Conwood, and Church & Dwight Co., Inc., supra note Error! Bookmark not defined.
specific computer software, training of employees that is particular to the supplier, and so forth).

4.4 The Implications for antitrust policy

Our results have several antitrust implications. The first is with regard to the use of slotting allowances – the fees that retailers, especially supermarkets and drugstores, ask from suppliers, usually as compensation for the retailer’s shelf space. Our results imply that slotting allowances may be more anticompetitive than the current economic literature predicts.

Note that ”slotting allowances,” in our context, include any fixed payment the supplier pays a retailer, regardless of its purpose. It need not be in exchange for shelf space, as traditionally predicted. Indeed, in practice, fees suppliers pay retailers, not in the form of per unit discounts, are paid for an array of reasons. Interestingly, slotting allowances paid to supermarket chains are a widespread phenomenon. According to analysts, American retailers make more than $18 billion in slotting allowances each year. In the UK, it is estimated that the big four supermarkets receive more in payments from their suppliers than they make in operating profits, and in Australia, it has been reported that growing supplier rebates have boosted food retailers’ profit margins by an average of 2.5 percentage points, to 5.7%, over the past five years. It was further reported that this phenomenon is not associated with low retail prices (The Economist, 2015).12 An EC study examining slotting allowances in the different European member states reports over 500 kinds of fees paid by suppliers to retailers, in addition to merely paying for shelf space.13 Under US case law, to date, slotting allowances have been rarely condemned, under the rule of reason, and only to the extent that they are paid in exchange for dominating retailers’ shelf space in a way that is likely to exclude rival suppliers.14 In our context, by contrast, the harm to competition stems from the mere payment of fixed fees by a dominant supplier to retailers. As long as the dominant supplier maintains its dominance in some way, the fees themselves need not have any exclusionary effect on rival suppliers for them to harm competition. It is not their exclusionary nature which harms competition in our

12 Notably, The Economist (2015) also reports that Walmart, known for heavy discount pricing, does not collect slotting allowances from suppliers.
13 These “excuses” include fees in consideration for promotion or advertising, or introductory allowances (see, e.g., FTC (2003)), listing fees, contributions for new store openings or store refurbishments, end of period bonuses, mergers and acquisitions, reimbursement of expenditures, and so forth. An EC study examining slotting allowances in the different European member states reports that over 500 different types of payments paid by suppliers to retailers were used (See Stichele, Vander and Young (2008)).
model, but rather the fact that they serve as a “prize” the supplier is willing to pay retailers in exchange for retailers’ adherence to the collusive scheme.\textsuperscript{15}

Interestingly, the European Commission’s guidelines on vertical restraints briefly identify that slotting allowances may facilitate downstream collusion.\textsuperscript{16} The guidelines do not deal, however, with the fact that according to the economic literature to date, one retailer needs to observe its rival’s contract with the supplier in order for slotting allowances to facilitate downstream collusion. Corollary 2 shows that slotting allowances can be anti-competitive even in the common case when contracts between suppliers and retailers are secret. Usually, a retailer cannot observe its rivals’ contracts with the supplier. After all, exchange of information among retailers competing in a relevant market regarding their commercial terms with a common supplier would most probably be condemned as an antitrust violation.\textsuperscript{17} We show that even though each retailer cannot observe the contract between the supplier and the competing retailer, retailers know that the supplier observes both contracts and has an incentive to maintain vertical collusion. Therefore, a retailer cannot profitably convince the supplier to accept a contract that motivates the retailer (and the supplier) to deviate from the collusive equilibrium.

Corollary 1 and proposition 1 also indicate that the anti-competitive effect of slotting allowances is not necessarily related to their size. When $\delta$ is close to zero, even though firms are very shortsighted such that it should be very difficult for them to maintain collusion, still a small slotting allowance is enough to maintain the collusive equilibrium. As firms care more about future profits, even though it becomes easier for them to collude, the size of slotting allowances actually increases. Then, after a certain threshold of $\delta$, the easier it becomes to sustain collusion (as $\delta$ increases), the size of the slotting allowances decreases again. This result indicates that antitrust authorities cannot necessarily infer the potential anti-competitive effect of slotting allowances from their mere magnitude.

In some cases, slotting allowances are paid by suppliers as compensation for intense competition among retailers over selling the supplier’s brand.\textsuperscript{18} Our results imply that such scenarios may deserve more lenient antitrust treatment, provided that the claim of compensation

\textsuperscript{15} As we show in section 4.3 above, competition among suppliers may dissipate the anticompetitive effect of slotting allowances. Hence, in the case of slotting allowances that are used to induce all retailers to buy exclusively from one supplier, their anticompetitive effect is exacerbated.
\textsuperscript{16} See European Commission (2012). In the EU, slotting allowances, like most vertical restraints, enjoy a safe harbor if both the supplier’s and each retailer’s market share is below 30%. See European Commission (2010). Some of the member states at the EU included strict prohibitions of slotting allowances, such as France (see Article L-442-6 of the French Code de Commerce); the UK (see GSCOP, Part 5, 12) and a proposed prohibition in Ireland (see Lianos 2010). A similar prohibition exists in Israel (The Law for the Promotion of Competition in the Food Sector, 2014), and Poland banned the practice in 1993 (see The Economist, 2015). In addition, Chinese authorities have challenged retailers for charging excessive slotting allowances in 2011 (The Economist, 2015).
\textsuperscript{17} See sources cited supra note 1.
\textsuperscript{18} See, e.g., Moulds (2015).
for intense competition is not a sham. In our framework, during or after a price war between retailers, when collusion collapses, slotting allowances are no longer used (see Lemma 1). On the contrary, when vertical collusion collapses, the supplier stops paying retailers slotting allowances in our framework, in order to punish retailers for not adhering to the collusive scheme. Conversely, if slotting allowances are paid under different circumstances, they should raise particular suspicion. Our results imply that, at least in the case where retailers purchase from only one supplier in a relevant supply market, and absent compelling pro-competitive motivations for using slotting allowances, they should be treated with high scrutiny: not only do they raise the probability of sustaining collusion, but their existence implies that they actually enabled collusion. Recall from Lemma 1 that the competitive equilibrium benchmark cannot involve slotting allowances, and from Corollary 1 that for \( \delta > 1/2 \) (where even horizontal collusion is sustainable, without the supplier's participation) there are no slotting allowances. Hence, had collusion not been sustainable, or alternatively, had horizontal collusion been sustainable without slotting allowances, the model predicts that there would not have been any slotting allowances in equilibrium.

Note that for slotting allowances to facilitate vertical collusion, they must take the form of fixed payments, rather than mere per-unit discounts. A per unit discount granted by the supplier to a retailer would ruin the collusion-facilitating nature of the scheme, since it is the elevated wholesale price per-unit that deters retailers from deviating from collusion.

All of the policy implications above concern a monopolistic supplier. Our results in section 4.3 and corollary 3, however, imply that when the supplier faces aggressive competition from other suppliers, slotting allowances can no longer facilitate collusion. This justifies more lenient antitrust treatment of slotting allowances in such cases.

Our results imply that the anticompetitive effect of slotting allowances in facilitating vertical collusion stems from concentration in the supplier's market, rather than in the retailers' market. It is straightforward to extend the model to any finite number of retailers and obtain a collusive equilibrium, as long as there is a monopoly supplier. Hence an even extremely competitive structure of the retail market cannot serve as a defense for slotting allowances paid by a dominant supplier.

As shown, even a period by period attempt by retailers to promise the dominant supplier to buy exclusively from it collapses for low discount factors. Only an extreme form of exclusive dealing, where all retailers make a long term commitment, through some external mechanism, to buying from the same supplier, coupled by slotting allowances, can facilitate vertical collusion.
Our results also have more general policy implications with regard to vertical restraints that help elimination downstream price competition, such as minimum resale price maintenance, suggested retail prices, or exclusive territories. According to the “Chicago School” approach, such vertical restraints should be treated leniently by antitrust authorities: Given the wholesale price the supplier collects, and given that the supplier earns no fixed fees from the retailers (but, on the contrary, pays the retailers slotting allowances) we would expect the supplier to prefer intense retail competition. According to this reasoning, elimination of downstream competition only harms the supplier, since it reduces the number of units sold, and the supplier’s only profits stem from the per-unit wholesale price. If the supplier does try to eliminate downstream competition, so the argument goes, it must be due to efficiencies in distribution rather than to harm consumers. According to this approach, a resale price dictated or suggested by the supplier would typically be considerably lower than the monopoly retail price.19 As we show, however, the supplier may strategically wish to sustain vertical collusion, in which the monopoly retail price is charged by retailers. This is because the supplier too reaps some of the profits from sustaining the monopoly price, and manages to charge a higher wholesale price than he would have been able to charge absent the collusive scheme. In our model, such collusion is achieved tacitly, and the supplier assists it via the wholesale price he collects and the slotting allowances he pays. But in the real world, the supplier may well attempt to make sure the vertical collusion scheme is sustainable, by using more intrusive vertical restraints, such as minimum resale price maintenance, exclusive territories, or suggested resale prices. In such occasions, our results imply that the above-mentioned lenient approach may not be justified, and that the retail price dictated or suggested by the supplier may well be the monopoly retail price.

4.5 Alternative trigger strategy
The previous section shows that for \( \delta < 1/2 \) the supplier earns higher profits than \( \pi_c \) even though retailers have the bargaining power to make take-it-or-leave-it offers and can coordinate on their most profitable collusive equilibrium. This result is driven by the assumption that if \( R_i \) observes that \( R_j \) didn't carry the product, \( R_i \) interprets it as a deviation by \( R_j \) and stops collusion. Such a trigger strategy provides the supplier with bargaining power, because if the supplier rejects \( R_j \)'s equilibrium offer, \( R_i \) will not carry the product, and consequently \( R_i \) will stop

19 Marvel (1994), for example, argues that "manufacturers will not voluntarily enforce cartels for their dealers … a manufacturer has no more interest in inefficient distribution than do consumers … . Higher mark ups [for retailers] mean that the net-of-margin demand curve faced by the manufacturer is lower than need be. Lower demand curves are less profitable. If retailer price competition is suppressed, the manufacturer must anticipate some benefit to offset the adverse effects of the higher dealer margins that result." For similar arguments in the legal and economic literature see, e.g., Bork (1978); Posner (1976); (1981); Easterbrook (1984); Telser (1960); Katz (1989); and Taussig (1916).
collusion. In this subsection we ask whether retailers can earn higher profits by using a softer trigger strategy, which removes the bite from the supplier's ability to stop vertical collusion by rejecting a retailer’s offer.

Suppose that whenever \( R_i \) observes that \( R_j \) didn’t carry the product, \( R_i \) interprets it as a deviation by the supplier rather than by \( R_j \) and continues with the collusive equilibrium. \( R_i \) stops offering the collusive contract only if \( R_j \) carried the product in the previous period but charged a different price than \( p^* \). Under such a trigger strategy, the supplier’s decision whether to accept a retailer’s offer no longer affects future collusion. Supposedly, retailers’ benefit from this alternative trigger strategy is that it may help them obtain a higher share of the monopoly profits. The disadvantage of this strategy for retailers, however, is that it increases their profit from defecting from the collusive equilibrium at the first stage of every period, as now the supplier cannot punish a retailer who did so.

With this alternative trigger strategy, condition (2) is still necessary to support a collusive equilibrium, because this condition prevents \( R_i \) from defecting from collusion in the second stage of the period. Turning to the supplier’s participation constraint, given that both retailers offer the equilibrium collusive contracts, the supplier’s decision on whether to accept both of them or just one is not going to affect the future. Hence the supplier’s participation constraint could be written as:

\[
Q^w + 2T^* = Q^w + T^*,
\]

where the left-hand-side is the supplier's profit from accepting the two equilibrium contracts and the right-hand-side is the supplier's profit from accepting only one of them. Neither of the supplier’s decisions affects collusion, and hence the supplier does not sacrifice any of his own future collusive profits by rejecting an equilibrium contract. This condition requires that \( T^* = 0 \). However, the proof of Corollary 2 showed that (2) cannot hold if \( T^* \geq 0 \) and \( \delta < \frac{1}{2} \), implying that this alternative trigger strategy cannot maintain a collusive equilibrium.

**Corollary 4:** Suppose that \( \delta < 1/2 \) and retailers do not stop collusion if they observe that the supplier accepted only one of the contract offers. Then, there are no contracts \((w^*, T^*)\) that can maintain a collusive equilibrium.

Corollary 4 shows that the retailers need the supplier’s participation to sustain collusion and they need to share future collusive profits with the supplier for collusion to be sustainable at low values of \( \delta \). If the supplier’s role in the collusive scheme is eliminated, there is no vertical contract in which collusion is sustainable. Retailers must charge slotting allowances for them to be willing to collude for low values of \( \delta \). But if retailers do not share collusive profits with
the supplier, and retailers ask for slotting allowances, the supplier always rejects one of the retailer’s offers.

Consider now the case where $\delta > 1/2$. In the collusive equilibrium that we defined in Proposition 1, for such discount factors, the supplier earns only its reservation profit, $\pi^C$. Hence retailers cannot do better by adopting an alternative trigger strategy.

5. Differentiated retailers

This section considers horizontally differentiated retailers. The main conclusion of the section is that secret vertical contracts can facilitate vertical collusion even when retailers are differentiated. However, unlike the case of homogeneous retails, differentiated retailers and their joint supplier cannot maintain collusion for all values of $\delta$. As retailers become closer substitutes, vertical contracts that include slotting allowances facilitate collusion for a larger range of discount factors.

In order to investigate how the degree of differentiation between the two retailers affects our results, consider a representative consumer with the utility function:

$$U(q_1, q_2) = \sum_{i=1}^{2} \left( q_i - \frac{1}{\sigma} q_i^2 \right) - \sigma q_1 q_2 - \sum_{i=1}^{2} (q_i p_i),$$

where $q_i$ and $p_i$ are the price and quantity of $R_i$, and $\sigma \in (0, 1)$ measures the degree of horizontal differentiation between the two retailers. When $\sigma = 0$, the two retailers are monopolies and they become closer substitutes as $\sigma$ is closer to 1. Differentiating (10) with respect to $q_1$ and $q_2$ yields the demand function facing $R_i$:

$$q_i(p_i, p_j) = \frac{1}{1 + \sigma} - \frac{1}{1 - \sigma^2} p_i + \frac{\sigma}{1 - \sigma^2} p_j.$$

Consider first the benchmark case of horizontal collusion between retailers that behave as two competing firms that can obtain the input at marginal costs 0. We ask under which values of $\delta$ such collusion is sustainable. The collusive prices that maximize the monopoly profit, $p_1 q_1(q_1, q_2) + p_2 q_2(q_2, q_1)$ are $p_1 = p_2 = p^* = 1/2$ which yields the monopoly quantity and profit of:

$$q^* \equiv q_1(p^*, p^*) = \frac{1}{2(1 + \sigma)}, \quad \pi^* \equiv p^* q^* = \frac{1}{4(1 + \sigma)}.$$

The competitive price of firm $i$ maximizes $p_i q_i(p_i, p_j)$ given $p_j$. Hence, the competitive price, quantity and profit are
\[
p^C = 1 - \frac{1}{2 - \sigma}, \quad q^C = q_i(p, p^C, C) = \frac{1}{2 + \sigma - \sigma^2}, \quad \pi^C = p^C q^C = \frac{1 - \sigma}{(2 - \sigma^2)(1 + \sigma)}.
\]

Notice that as \( \sigma \to 1 \), the two products become close substitutes and therefore \( p^C \to 0 \) and \( \pi^C \to 0 \).

When \( R_i \) sets the collusive price, \( p^* \), while \( R_j \) deviates from collusion, then \( R_i \) sets \( p_i \) to maximize \( p_i q_i(p, p^*) \). Let \( p(p) \) denote \( R_i \)'s best-response to \( p_j \). Hence, the deviating price, quantity and profit are:

\[
p(p^*) = \frac{1}{4} (2 - \sigma), \quad q(p(p^*), p^*) = \frac{2 - \sigma}{(1 - \sigma^2)}, \quad \pi^D = p(p^*) q(p(p^*), p^*) = \frac{1}{16} \left[ \frac{2 - \sigma}{(1 - \sigma^2)} \right]^2.
\]

In what follows, we make the simplifying assumption that \( 0 < \sigma < \sqrt{3} - 1 \approx 0.73 \). When \( \sigma \) is close to 1, the two retailers are close substitutes and therefore when a retailer deviates from the collusive equilibrium, the retailer finds it optimal to fully dominate the market. Focusing on small enough values of \( \sigma \) ensures that \( q(p^*, q(p^*)) > 0 \), such that when \( R_i \) deviates from collusion, \( R_j \) still sells a positive quantity.

Horizontal collusion, without vertical contracts, is therefore possible if:

\[
\frac{\pi^*}{1 - \delta} > \pi^D + \delta \pi^C \iff \delta > \delta^C = \frac{(2 - \sigma)^2}{8 - \sigma(8 - \sigma)}.
\]

It is straightforward to show that for \( 0 < \sigma < 0.73 \), and \( 1/2 < \delta^C < 1 \), \( \delta^C \) is increasing with \( \sigma \).

Next, we turn to the case where retailers have secret vertical contracts with the supplier and we ask whether the parties can maintain vertical collusion for \( \delta < \delta^C \). We construct a collusive equilibrium in which in every period the two retailers offer the supplier the first stage the secret contract \( (w^*, T^*) \) that the supplier accepts, and then in the second stage the two retailers set the collusive price \( p^* \). As in our base model, suppose that any observable deviation at period \( t \) triggers the perfectly competitive equilibrium from period \( t + 1 \) onwards. In this section we focus on the competitive equilibrium in which each retailer offers the supplier \( w_i = T_i = 0 \) and then the two retailers charge \( p^C \) and earn \( \pi^C \) while the supplier earns \( \pi^C = 0 \).

The collusive contract has to satisfy two conditions. The first condition is that once retailers offered a contract \( (w^*, T^*) \) that the supplier accepted, \( R_i \) indeed plays in stage 2 the collusive

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20 Our approach is consistent with O’Brien and Shaffer (1992) who find that when two differentiated retailers sign secret vertical contracts with a joint supplier, there is a unique "negotiation proof" contract equilibrium in which retailers set \( w = T = 0 \) and then charge the competitive prices.
price, \( p^* \), instead of deviating to \( R \)'s short-run best response to \( p^* \). Let \( p_i(p^*; w^*) \) denote the \( p_i \) that maximizes \( R \)'s profit given \( p_j = p^* \) and given \( w^* \), \((p_i - w^*)q_i(p, p^*)\), where:

\[
p_i(p^*; w^*) = \frac{1}{4}(2 - \sigma + 2w^*).
\]

The first necessary condition is therefore:

\[
(p^* - w^*)q^* + \frac{\delta}{1-\delta}((p^* - w^*)q^* - T^*) \geq \left( p_i(p^*; w^*) - w^* \right)q_i(p_i(p^*; w^*), p^*) + \frac{\delta}{1-\delta}\pi^C_R,
\]

where the left hand side is \( R \)'s profit from maintaining collusion and the right hand side is \( R \)'s profit from deviating. Notice that (12) is the equivalent of condition (2) for the case where retailers are differentiated.

Next, we move to the second condition regarding the collusive contract, which involves the supplier's participation constraint. Suppose that at the beginning of a certain period, both retailers offered the supplier the collusive contract \((w^*, T^*)\). If the supplier accepts both offers, the supplier earns in the current period \( 2(w^*q^* + T^*) \) and collusion continues to the next period. Suppose, however, that the supplier decides to deviate from collusion by rejecting one of the offers, say, the offer of \( R_2 \). \( R_1 \) cannot observe this deviation in the second stage of the period, and will therefore set \( p_1 = p^* \). Let \( q(p^*, \infty) \) denote the quantity that \( R_1 \) sells when it charges \( p_1 = p^* \) and consumers cannot buy from \( R_2 \). We can solve for \( q(p^*, \infty) \) by substituting \( q_2 = 0 \) and \( p_1 = p^* \) into the utility of the representative consumer in (10) and differentiating with respect to \( q_1 \). Hence, we obtain that \( q(p^*, \infty) = 1/2 \). The supplier earns \( w^*q(p^*, \infty) + T^* \) from this deviation in the current period, but then collusion stops in all future periods. The supplier's participation constraint is therefore:

\[
\frac{2(w^*q^* + T^*)}{1-\delta} = w^*q(p^*, \infty) + T^*.
\]

Condition (13) is the equivalent of condition (3) for the case where retailers are differentiated.\(^{22}\)

Solving (13) for \( T^* \) yields:

\[
T^* = \left[ \frac{1-\sigma + \delta(1+\sigma)}{2(1+\delta)(1+\sigma)} \right] w^*.
\]

\(^{21}\) By our assumption that \( \sigma \) is not too high, when \( R \), deviates from the collusive price, \( R \), does not fully monopolize the market.

\(^{22}\) Notice that the right-hand-side of (16) is positive, because \( q(p^*, \infty) > q^* \) and \( w^*q^* + T^* > 0 \).
The term in the squared brackets in (14) is positive, implying that $T^* < 0$ whenever $w^* > 0$. We can rewrite the supplier and retailers’ one-period profits as a function of $w^*$ as:

$$
\pi_R(w^*) = \frac{1}{4(1+\sigma)} - \frac{(1-\delta)\sigma}{2(1+\sigma)(1+\delta)} w^*, \quad \pi_S(w^*) = \frac{(1-\delta)\sigma}{1+\delta+\sigma+\delta\sigma} w^*.
$$

As in our base model, $\pi_R(w^*)$ is decreasing in $w^*$ while $\pi_S(w^*)$ is increasing in $w^*$ and $\pi_S(w^*) > 0$ if and only if $w^* > 0$

It is possible to extend the proof of proposition 1 and show that conditions (12) and (13) are sufficient for a collusive equilibrium when retailers are differentiated. In the retailers’ most profitable collusive equilibrium, retailers choose the lowest possible $w^* > 0$ that satisfies (12) and (13). Substituting (14) into (12) and rearranging, (12) becomes:

$$
\left[\frac{\pi^*}{1-\delta} - \frac{\pi_R^D}{1-\delta} + \frac{\delta}{1-\delta} \pi_R^C\right] + \gamma w^* (\Omega - w^*) > 0, \quad (16)
$$

where

$$
\Omega \equiv \frac{\sigma(1-\delta+2\delta\sigma)}{1+\sigma} > 0, \quad \gamma \equiv \frac{1}{4(1-\sigma^2)} > 0.
$$

Comparing (16) with the inequality that defines $\delta^C$ in (11) yields that when $w^* = 0$, the term in the squared brackets in (16) is positive if and only if $\delta > \delta^C$ and consequently so is (16). However, when $w^* > 0$, it is possible to satisfy (16) even when the term in the squared brackets is negative, i.e., for $\delta < \delta^C$, as long as

$$
\frac{\pi^*}{1-\delta} - \frac{\pi_R^D}{1-\delta} + \frac{\delta}{1-\delta} \pi_R^C > -\frac{\gamma \Omega}{4}.
$$

(17)

In such a case, $w^* > 0$ must involve $T^* < 0$, because of (14). Let $\delta^*$ denote the solution to (17) in equality. The above discussion yields the following result:

**Proposition 2:** Suppose that retailers are horizontally differentiated according the representative consumer’s preferences in (10) and $\sigma \in (0, 0.73]$. Then,

(i) For $\delta \in [0, \delta^*]$ there is no collusive equilibrium;

(ii) For $\delta \in [\delta^*, \delta^C]$ there are collusive equilibria. The retailers’ most profitable collusive equilibrium involves $w^* > 0$ and $T^* < 0$;

(iii) For $\delta \in [\delta^C, 1]$ there are collusive equilibria. The retailers’ most profitable collusive equilibrium involves $w^* = 0$ and $T^* = 0$. 

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Proposition 2 shows that when retailers are differentiated, vertical collusion is sustainable for values of $\delta$ under which horizontal collusion is impossible. Figure 2 plots $\delta^C$ and $\delta^*$ as a function of $\sigma$ for $\sigma \in (0, 0.73]$. In the region below $\delta^*$, there is no collusive equilibrium and the two retailers earn the competitive profit $\pi_R^C$ while the supplier earns 0. In the region between $\delta^*$ and $\delta^C$, firms can maintain a collusive equilibrium with $w^* > 0$ and $T^* < 0$, and all three firms earn positive profits. It is possible to show that as in our base model, in this region, the retailers' profits increase with $\delta$ while the supplier's profit decreases with $\delta$. In the region above $\delta^*$, the two retailers can engage in horizontal collusion, without including the supplier in the collusive scheme. In this region, retailers set $w^* = T^* = 0$, collude on the monopoly price, and each retailer earns half of the collusive profit while the supplier earns 0.

Recall that in the base model with homogeneous retailers, the retailers and the supplier can maintain a collusive equilibrium for all positive values of $\delta$. Proposition 2 shows, by contrast, that when retailers are differentiated, the three firms cannot maintain a collusive equilibrium when $\delta$ is sufficiently small. As retailers become closer substitutes, however, (i.e., $\sigma$ increases), $\delta^C$ increases, $\delta^*$ decreases and consequently the region in which vertical collusion is more sustainable than horizontal collusion increases. The intuition for these two results is that retailers' differentiation makes it costly for the supplier to reject a deviating contract offer from a retailer, because by doing so the supplier does not gain access to certain consumers. This feature decreases the supplier's market power, which in turn decreases its ability to police the two retailers' adherence to the collusive equilibrium. This is why as retailers become more differentiated, collusion becomes harder to sustain, even when retailers use vertical contracts.

![Figure 2: The cutoffs $\delta^C$ and $\delta^*$ as a function of $\sigma$](image-url)
6. Conclusion

We consider dynamic collusion involving retailers and their joint supplier. Our model of vertical collusion has two main features. First, all three firms equally care about the future and they all participate in the collusive scheme. Second, vertical contracts are secret: a retailer cannot observe the bilateral contracting between the competing retailer and the supplier.

Retailers gain from vertical collusion, because it enables them to charge the monopoly retail price even for discount factors that would not have enabled ordinary horizontal collusion among them, and they receive slotting allowances from the supplier as a prize for participating in the collusive scheme. The supplier gains from vertical collusion, because he collects a higher wholesale price and makes a higher profit than absent vertical collusion. This occurs even when retailers have all the bargaining power, where the supplier’s difficulty in receiving a high wholesale price is at its peak. Also, it occurs despite the fact retailers are too impatient to sustain horizontal collusion, and despite the fact the supplier is as impatient as retailers are.

Finally, fierce upstream competition causes vertical collusion to break down. This result implies that when retailers face fierce upstream competition, implementing a vertical collusive scheme requires the retailers to be able to commit to dealing exclusively with the same supplier. In real-life, such a commitment may involve an exclusive dealing contract that retailers may sign with a joint supplier prior to the vertical collusion scheme. Alternatively, retailers can make relationship-specific investments with regard to a joint supplier, such as establishing order and payment procedures with the supplier, which will make it more costly for retailers to deal with an alternative supplier. The results of our paper imply that such practices can facilitate vertical collusion, which in turn harms consumers and reduces social welfare.
Appendix
Below are the proofs of Lemma 1, Proposition 1 and Corollary 2.

Proof of Lemma 1:
We will proceed in two steps. In the first step, we will show that if (1) does not hold then $R_i$ finds it optimal to deviate to a contract that motivates the supplier to reject the contract of $R_i$, but this deviation is impossible if (1) holds. In the second step we show that $R_i$ cannot profitably deviate to a contract that does not motivate the supplier to reject the contract of $R_i$.

We first show that if (1) does not hold, $R_i$ can make a profitable deviation. Since $p(w) > w$ and $pQ(p)$ is concave in $p$:
\[
\max_{w_i} \{ w_i Q(p(w_i)) \} < \max_{w^C} \{ w^C Q(w^C) \},
\]
\[
\max_{w_i} \{ w_i Q(p(w_i)) \} > w^C Q(w^C) |_{w^C = p^*}, \quad \text{and:} \quad \max_{w_i} \{ w_i Q(p(w_i)) \} < w^C Q(w^C) |_{w^C = p^*},
\]

implying that there is a $w_i$ such that (1) holds for $w^C \in [w_L, p^*]$ and does not hold otherwise, where $w_L > 0$. Suppose that (1) does not hold. Then $R_i$ can deviate to $(T_i, w_i)$ such that $w_i Q(p(w_i)) > w^C Q(w^C)$. If the supplier accepts the contract, it rationally (for both the supplier and $R_i$) to expect that the supplier does not accept the contract of $R_i$ and that $R_i$ sets $p(w_i)$. Given these expectations, the supplier agrees to the deviating contract if $w_i Q(p(w_i)) + T_i \geq w^C Q(w^C)$, or $T_i = w^C Q(w^C) - w_i Q(p(w_i))$. $R_i$ earns from this deviation:
\[
(p(w_i) - w_i)Q(p(w_i)) - T_i = p(w_i)Q(p(w_i)) - w^C Q(w^C) > w_i Q(p(w_i)) - w^C Q(w^C) > 0,
\]
where the first inequality follows because $p(w_i) > w_i$ and the second inequality follows because whenever (1) does not hold it is possible to find $w_i$ such that $w_i Q(p(w_i)) > w^C Q(w^C)$. Since $R_i$ earns in equilibrium 0, $R_i$ finds it optimal to deviate. Now suppose that (1) holds. Then, there is no $w_i$ that ensures that the supplier does not accept the contract of $R_i$.

Next, we turn to the second step of showing that $R_i$ cannot make a profitable deviation when $R_i$ expects that the supplier accepts the equilibrium contract of $R_i$. Suppose that $R_i$ deviates to $(T_i, w_i) \neq (0, w^C)$ such that if the supplier accepts the deviation, the supplier continues to play the equilibrium strategy of accepting the contract offer of $R_i$, $(0, w^C)$. $R_i$ therefore expects that $R_i$ will be active in the market and will set $p^C = w^C$. The deviation can be profitable to $R_i$ only if $w_i < w^C$, such that $R_i$ can charge in stage 2 a price slightly lower than $w^C$ and dominate the market. To convince the supplier to accept the deviating contract, $R_i$ charges $T_i$ such that the supplier is indifferent between accepting both offers and accepting just the equilibrium offer of $R_i$: $w_i Q(w^C) + T_i \geq w^C Q(w^C)$, or $T_i \geq (w^C - w_i) Q(w^C)$. But then $R_i$ earns at most $(w^C - w_i) Q(w^C)$.
– $T_i \leq 0$. We therefore have that $R_i$ cannot offer a profitable deviation from the equilibrium $(0, w^C)$ if $R_i$ believes that the supplier accepts the equilibrium contract of $R_j$.

**Proof of Proposition 1:**

We will move in three steps. In the first step we solve for the set of $(w^*, T^*)$ that satisfy (2), (3) and $\pi_S(w^*) \geq \pi_C$. In the second step we show that the set of $(w^*, T^*)$ ensures that $R_i$ cannot profitably deviate to $(w_i, T_i) \neq (w^*, T^*)$. We will assume wary beliefs such that $R_i$ expects that the supplier accepts the contract of $R_j$ only if it is profitable for the supplier to do so. In Lemma A1 we will show that if the supplier expects that by accepting both $R_i$'s deviating offer and $R_j$'s offer $R_i$ will defect from collusion, then the supplier will not accept both offers to begin with. This implies that if the supplier accepts a deviating offer by $R_i$ in the first stage of a certain period, the supplier accepts the equilibrium offer of $R_j$ only if the supplier expects that $R_i$ will maintain collusion at the second stage. We can therefore restrict attention to the following two cases that we examine in Lemma A2 and Lemma A3. Lemma A2 shows that $R_i$ cannot profitably deviate to a contract $(w_i, T_i) \neq (w^*, T^*)$ such that if the supplier accepts the deviating offer of $R_i$, the supplier also accepts and the equilibrium offer of $R_j$ and then $R_i$ continues to maintain collusion. We do not impose constraints on the set of possible $(w_i, T_i)$ that ensures that $R_i$ indeed maintains collusion given the deviating contract because we show that even the unconstrained set of $(w_i, T_i)$ is never profitable for $R_i$. In Lemma A3 we show that $R_i$ cannot profitably deviate to a contract $(w_i, T_i) \neq (w^*, T^*)$ such that if the supplier accepts the deviating offer of $R_i$, the supplier does not accept and the equilibrium offer of $R_j$. Again we show that this holds for any $(w_i, T_i)$ and therefore we do not need to impose restrictions on the set of possible $(w_i, T_i)$ that support such beliefs. In the third step we solve for the $(w^*, T^*)$ that maximizes the retailers’ profits subject to (2), (3) and $\pi_S(w^*) \geq \pi_C$.

Starting with the first step, extracting $T^*$ from (3) yields:

$$T^*(w^*) = \frac{\delta (\pi_C - Q^* w^*)}{(1 + \delta)}.$$  \hspace{1cm} (A-1)

Substituting (A-1) into (3) we can rewrite (2) as:

$$w^* > p^* - \frac{2\delta^2 (p^* Q^* - \pi_C)}{(1 - \delta) Q^*}.$$  \hspace{1cm} (A-2)

Substituting (A-1) into $\pi_S(w^*)$ we have:

$$\pi_S(w^*) = \frac{1 - \delta}{1 + \delta} \frac{2\delta}{1 + \delta} \pi_C > \pi_C \quad \Leftrightarrow \quad w^* > \frac{\pi_C}{Q^*}.$$  \hspace{1cm} (A-3)
Comparing the right-hand-sides of (A-2) and (A-3), the former is higher than the latter iff $\delta < 1/2$. We conclude that (2), (3) and $\pi_\delta(w^*) \geq \pi^C$ hold for any $T^*(w^*)$ defined by (A-1) and $w^*$, where:

$$w^* \geq w^E \equiv \begin{cases} p^* - \frac{2\delta^2(p^*Q^*-\pi^C)}{(1-\delta)Q^*}; & \delta \in [0, \frac{1}{2}]; \\ \pi^C/Q^*; & \delta \in [\frac{1}{2}, 1]. \end{cases}$$

Next, we turn to the second step of showing that the set of $w^* \geq w^E$ and $T^*(w^*)$ ensures that $R_i$ cannot profit from deviating to another $(w_i, T_i) \neq (w^*, T^*)$.

We first show that given that the supplier accepts a deviating offer by $R_i$, the supplier does not accept the equilibrium contract of $R_j$ if the deviating contract motivates $R_i$ to deviate from the collusive price in the second stage of the period.

**Lemma A1:** Suppose that in the first stage of a certain period $R_i$ offers a deviating contract $(w_i, T_i) \neq (w^*, T^*)$ that motivates $R_i$ to deviate from the collusive price at the second stage of the period. Under wary beliefs $R_i$ cannot rationally expect that if the supplier accepts $R_i$'s contract, the supplier also accepts the equilibrium contract offer of $R_j$.

**Proof:** We will show that for all $w^* \geq w^E$, $T^*(w^*) \leq 0$. Consequently, the supplier cannot make positive profits from $R_j$ by accepting both offers because the supplier expects that $R_j$ cannot make positive sales while accepting the offer of $R_i$ result in paying: $-T^*(w^*)$. To see why $T^*(w^*) \leq 0$, for $\delta < 1/2$: 

$$T^*(w^*) = \frac{\delta(\pi^C - Q^*w^*)}{1+\delta} \leq \frac{\delta(\pi^C - Q^*w^E)}{1+\delta} = -\frac{\delta}{1-\delta}(1-2\delta)(p^*Q^* - \pi^C) < 0,$$

where the first inequality follows because $w^* \geq w^E$ and the second inequality follows because $\delta < 1/2$ and $p^*Q^* > \pi^C$. For $\delta > 1/2$, $T^*(w^*) \leq T^*(w^E) = T^*(\pi^C/Q^*) = 0$.

Lemma A1 implies that if the supplier accepts a deviating offer by $R_i$ in the first stage of a certain period, the supplier accepts the equilibrium offer of $R_j$ only if the supplier expects that $R_i$ will maintain collusion at the second period. Below we show that if there is such a deviating contract, $(w_i, T_i) \neq (w^*, T^*)$, $R_i$ will not offer it. We do not impose constraints on $(w_i, T_i)$ that support these beliefs but show that given any $(w_i, T_i)$ that support these beliefs, the deviation is not profitable to $R_i$. 

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Lemma A2: Suppose that in the first stage of a certain period, \( R_i \) offers a deviating contract \((w_i, T_i) \neq (w^*, T^*) \) such that the supplier and \( R_i \) expects that if the supplier accepts the deviation, the supplier also accepts the offer of \( R_i \) and \( R_i \) maintains collusion in the second stage of this period. Than, \( R_i \) cannot profit from making such a deviation.

Proof: Suppose that the supplier and \( R_i \) have the common beliefs that if the supplier accepts the deviation, the supplier also accepts the offer of \( R_i \) and \( R_i \) maintains collusion. Whenever \( R_i \) makes this deviation, the supplier expects that \( R_i \) will set \( p^* \) in the current period and therefore \( R_j \) will not detect it. The supplier's profit from accepting the deviation depends on whether the supplier expects that \( R_i \) will offer in the next period the equilibrium contract or continue offering the deviating contract. We consider each possibility in turn. Suppose first that the supplier expects that \( R_i \) offers a one-period deviation, \((w_i, T_i)\), and will continue offering \((w^*, T^*)\) in all future periods. The supplier anticipates that by accepting this contract, this deviation will not be detected by \( R_j \) and therefore collusion is going to maintain in future periods. Therefore, the supplier accepts the deviation iff:

\[
 w^* Q^*/2 + T^* (w^*) + \frac{\delta}{1-\delta} \left( w^* Q^* + 2T^* (w^*) \right) > w^* Q^* + T^* (w^*) + \frac{\delta}{1-\delta} \pi_C,
\]

where the left-hand-side is the supplier's profit from accepting a one-period deviation given that doing so maintains the collusion equilibrium in all future periods and the right-hand-side is the supplier's profit from accepting \( R_i \)'s contract and stopping collusion. Substituting (A-1) into (A-4) and solving for \( T_i \), the supplier accepts the deviation if:

\[
 T_i > \frac{\delta}{1+\delta} \pi_C + \frac{1-\delta}{2(1+\delta)} Q^* w^* - \frac{Q^* w_i}{2}.
\]

\( R_i \) prefers making this one-period deviation if \( R_i \) earns higher one-period profit than the equilibrium profit. However, \( R_i \)'s profit from this deviation is:

\[
 (p^*-w_i) Q^*/2 - T_i < \frac{1}{2} \left( p^* \frac{1-\delta}{1+\delta} w^* \right) Q^* - \frac{\delta}{1+\delta} \pi_C = \pi_e(w^*).
\]

where the inequality follows from substituting (A-5) into \( T_i \) in (A-6). Notice that we only need to look at the one-period profit because if the supplier accepts the deviation then \( R_i \)'s future profits are \( \pi_e(w^*) \). We therefore have that \( R_i \) cannot profit from making the deviation. Suppose now that the supplier expects that \( R_i \)'s deviation is permanent. Now, the supplier agrees to the deviation if:
\[
\frac{w^* Q^*/2 + T^*(w^*) + w_i Q^*/2 + T_i}{1 - \delta} > \frac{w^* Q^* + T^*(w^*) + \delta}{1 - \delta} \pi^c,
\]
where the left-hand-side is the supplier's profit from accepting the deviation given that the supplier expects that the deviation is permanent and the right-hand-side is identical to (A-4).

The supplier agrees to the deviation if:

\[
T_i > \frac{\delta}{1 + \delta} \pi^c + \frac{1 - \delta}{2(1 + \delta)} Q^* w_i - Q^* w_j.
\]

(A-7)

\(R_i\)'s profit from making this deviation in the current and all future periods is:

\[
\left(\frac{p^* - w_i)Q^*/2 - T_i}{1 - \delta} \right) < \frac{1}{1 - \delta} \left( \frac{1}{2} \left( p^* - \frac{1 - \delta}{1 + \delta} w_i \right) Q^* - \frac{\delta}{1 + \delta} \pi^c \right) = \frac{\pi_k(w^*)}{1 - \delta},
\]

where the inequality follows from substituting \(T_i\) in (A-7) into (A-8). We therefore have that \(R_i\) cannot profitably make a permanent deviation to \((w_i, T_i)\) that motivates \(R_i\) to maintain collusion.

Next we turn to the last deviating option for \(R_i\), which is to deviate to a contract such that if the supplier accepts the deviation, the supplier does not find it profitable to accept the equilibrium contract of \(R_j\). In the following lemma we show that if there is such a deviating contract, \((w_i, T_i) \neq (w^*, T^*)\), \(R_i\) will not offer it. As with Lemma A2, we do not impose constraints on the set of \((w_i, T_i)\) that support these beliefs but show that given any unconstrained set of \((w_i, T_i)\) that support these beliefs, the deviation is not profitable to \(R_i\).

**Lemma A3:** Suppose that in the first stage of a certain period \(R_i\) offers a deviating contract \((w_i, T_i) \neq (w^*, T^*)\) such that if the supplier accepts the contract, the supplier does not accept offer of \(R_j\). Than, \(R_i\) cannot profit from making such a deviation.

Suppose that \(R_i\) deviates to \((w_i, T_i)\) given the beliefs that if the supplier accepts the deviation, the supplier rejects the contract of \(R_j\). The supplier accepts the deviation if \(w_i Q(p(w_i)) + T_i > w^* Q^* + T^*(w^*)\), or:

\[
T_i > w^* Q^* - w_i Q(p(w_i)) + T^*(w^*).
\]

(A-9)

\(R_i\) earns from this deviation:

\[
(p(w_i) - w_i)Q(p(w_i)) - T_i
< p(w_i)Q(p(w_i)) - w^* Q^* - T^*(w)
= Q\left( p^* - \frac{w^*}{1 + \delta} \right) - \frac{\delta}{1 + \delta} \pi^c,
\]

(A-10)

\[
< \frac{\pi_k(w^*)}{1 - \delta}
\]

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where the first inequality follows from substituting the right-hand-side of (A-9), the equality follows from substituting (A-1) and the second inequality holds iff $w^* > w^E$, implying that this deviation is not profitable for $R_i$ for $w^* > w^E$.

Finally, we turn to the last step of solving for the collusive equilibrium that maximizes the retailers’ profits subject to (2), (3) and $\pi_S(w^*) > \pi_C$. From (4), $\pi_R(w^*)$ is decreasing with $w^*$ and therefore the most profitable equilibrium is $w^* = w^E$. Substituting $w^E$ into (A-1) yields (5) and (6).

**Proof of Corollary 2:**
Suppose that retailers have choose a collusive equilibrium subject to the constraint $T(w^*) \geq 0$. From (A-1), $T(w^*) \geq 0$ requires $w^* \leq \pi^E Q^*$. However, (3) requires that $w^* > w^E > \pi^E Q^*$ where the last inequality holds for all $\delta < 1/2$. Therefore, it is impossible to obtain a collusive equilibrium with $T^*(w^*) > 0$ for $\delta < 1/2$. 
References


[47] The Economist. 2015. “Retailers and supplier rebates: Buying up the shelves, Supplier rebates are at the heart of some supermarket chains’ woes, June 20th.