UNDERSTANDING PROPORTIONAL LIABILITY

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Abstract

In torts, courts often face uncertainty regarding the injurer’s behavior, factual causation and the magnitude of harm. This Paper analyzes how two decision rules – proportional liability and preponderance of the evidence – affect care and judicial errors in a setting of unilateral accidents under a negligence regime. It shows that proportional liability is socially preferable when injurers and courts share a similar uncertainty (e.g., when uncertainty is about factual causation or the magnitude of harm), and that it minimizes court errors when uncertainty concerns a continuous variable (e.g., magnitude of harm). Following these insights, I argue that applying proportional liability to damages calculation and preponderance of the evidence to the determination of negligence is both error-minimizing and creates better incentives for the injurer, while I identify a tradeoff between the two goals when uncertainty is about factual causation.

1. Introduction

An injurer is liable in negligence if and only if her action was negligent, the plaintiff suffered harm, and that harm was caused by her negligent act. Together, these requirements represent the factual elements of liability under negligence law: negligence, harm, and factual causation.¹

Making factual determinations from the evidence presented is often an inaccurate and costly process and accordingly also prone to errors. Decision makers aware of the risk of error can arrange the facts in a hierarchy, based on their probability. Under this view, rules of evidence have a dual purpose: to reduce the risk of errors by the court, and to allocate the residual risk between the parties. The second purpose is achieved by

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¹ There are several legal elements of tort liability, such as proximate causation and duty of care. However, since these elements are not factual, we cannot think of them in terms of probability. The determination of the standard of care is often viewed as a legal decision, but according to the Learned Hand formula, the standard is based solely on facts, albeit not only the facts of the case at hand. In that sense, errors in the determination of the standard of care are similar in structure to errors in comparing the level of actual care to the standard (Cooter and Ulen 2012, 221).
“decision rules”.  

Thus, factual determinations can be guided by two types of decision rules: “threshold rules” and “proportional liability rules” (“PLR”). Applying a threshold rule, the court weighs the evidence and awards full compensation whenever the probability that each of the factual requirements was met crosses a certain, predetermined threshold. In civil disputes this predetermined threshold is usually 50%, and the rule is known as the “preponderance of the evidence rule” (“PER”). Contrarily, courts applying PLR award damages equal to the plaintiff’s harm multiplied by the probability that the factual requirements were met.

To illustrate the difference between the two types of rules, consider the following example. Assume that the court finds that the plaintiff’s (victim’s) harm equals $100, and that the probability that the harm was caused by the defendant's negligent act is 40%. In these circumstances, if the court applies PLR, damages will be set at $40. If, however, the court adopts PER, it will dismiss the case because causation has not been proven beyond the preponderance of the evidence.

Legal scholars usually follow one of two rationales when analyzing which decision rule courts should apply: either for the sake of minimizing legal mistakes, or to create optimal incentives for future behavior. Following the first rationale, Kaye (1982) suggested that, compared to PLR, PER is better able to minimize errors, because it reduces the aggregated sum of erroneous allocation of damages awards – an idea later followed by other scholars (Levmore 1990). Similarly, while Stein (2005, 143–153) noted that Kaye’s argument only holds as long as false negatives are considered equally

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2 Stein (2005, 152-167) refers to the rules of evidence and civil procedure that are designed to reduce the likelihood that the court will make a mistake as “procedural rules,” and to rules that allocate the risk of a mistake between the parties as “decision rules.” Following Stein, I use the term “decision rules” here in the same manner.

3 Abramowicz (2001) suggested combining PLR and threshold rules: the court would use an all-or-nothing rule when the probabilities are very high or very low, and implement PLR in the interim. Abramowicz’s suggestion might be better than any one rule, but it is not currently applied. This article focuses on the efficiency of decision rules, given the available rules that have already been adopted, and therefore does not consider a mixed rule.

4 Theoretically, we can set an ad hoc threshold for each case. Kaplow (2013 ;2012) suggested setting a threshold on a case-by-case basis (or by categories of cases), based on the relative costs of false positive and false negative. In this article I am trying to examine current practices adopted by the courts. Since Kaplow’s Ideas have not yet been adopted, I assume here that PER is the only threshold decision rule available in civil disputes.

5 For a case where the court dismissed a case because the plaintiffs failed to prove causation beyond the preponderance of the evidence, see, e.g., Merrell Dow Pharmaceuticals v. Havner, 953 S.W.2d 707(Tex. Sup. J. 1997); Dumas v. Gooney, 235 Cal. App. 3d 1593 (1991).
harmful as false positives, he further mentioned that this is indeed the case in civil disputes.

Tort scholarship, on the other hand, usually follows the latter rationale of creating optimal ex-ante incentives for efficient behavior. Accordingly, many legal economists point to PLR as a more desirable rule when applied to circumstances of factual uncertainty, at least when it is applied to factual causation or calculation of damages, because, as several authors have shown, the incentives created by PLR are at least as efficient as (and sometimes more efficient than) those created by PER (Shavell 2004, 240–241, 254–256; Shavell 1987, 115–118, 131–132; Shavell 1985; Makdisi 1988; Kaplow and Shavell 1996; Rosenberg 1984; Bartsch 1996; Robinson 1985). Moreover, as Porat (2011) showed, not only is PLR more efficient, it is also consistent with how courts apply the “Hand Formula” when setting the standard of care.

In practice, courts are seemingly inconsistent in the decision rule they apply. When uncertainty is about the magnitude of harm, most jurisdictions in the U.S., as well as in England, Canada and Israel, apply PLR and award damages equal to the expected magnitude of harm. Conversely, in most circumstances of uncertainty about factual causation, courts adopt PER as the decision rule. A notable exception is circumstances in which causation is inherently hard to prove, where some jurisdictions again prefer PLR in certain instances. For example, factual causation is often difficult to prove in medical malpractice, because of the high levels of background risk, and in many cases the plaintiff is unable to prove causation. To resolve this problem, some jurisdictions today

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6 For U.S. law, see (Dobbs 2000, sec. 377); For the law in England, see (Wheat 2002, 175); Gregg v. Scott [2005] UKHL 2. In Andrews v. Grand & Toy Alberta Ltd, the court stated that although the court should determine that the causation requirement is not met when the probability is lower than 50%, it should take into account events with low probability when calculating damages. For Canadian law see Andrews v. Grand & Toy Alberta Ltd., [1978] 2 S.C.R. 229 (the Canadian supreme court called for full compensation of all probable future losses, based on their probability). For Israeli law, see CA 10064/02 Migdal Ins. v. Rim Abu Hanna, [2005] IsrSC (the Supreme Court decided that the value of lost earning capacity of a child should be equal to the mean salary, taking into account every possible income level).

7 The use of PLR is restricted to cases where the plaintiff suffers some sort of harm in the present, but the magnitude of the harm depends on future, uncertain, events. If the victim suffers only from the risk of harm in the future, some common law jurisdictions do not allow compensation. Porat and Stein (2011) showed that from an efficiency perspective, courts should allow plaintiffs to recover for future harms, in accordance with PLR. This, however, was not always the case. In the past, U.S. courts have demanded that the plaintiff prove all damage heads, even if the harm might materialize in the future, beyond the preponderance of the evidence (Cooper, 1972).

8 See Rewis v. United States, 503 F.2d 1202 (5th Cir.1974); Grody v. Tulin, 170 Conn. 443, 365 A.2d 1076 (1976); Gooding v. University Hospital Building, 445 So.2d 1015 (Fla.1984); Curry v. Summer, 136 Ill.App.3d 468, 91 Ill.Dec. 365, 483 N.E.2d 711 (1985); Walden v. Jones, 439 S.W.2d
adopt the doctrine of “lost chance of recovery,” allowing compensation to be based on the probability that the harm was caused by the doctor’s negligent behavior (i.e., a PLR type of rule). Finally, all (or almost all) jurisdictions apply PER when the uncertainty is about whether or not the injurer’s behavior was negligent. Interestingly, the incentives that would be created by applying PLR to these types of circumstances have not yet been addressed by law and economics scholars.

At first glance, then, courts appear to be inconsistent in choosing decision rules. It is this puzzling phenomenon that this article addresses, suggesting that only when we consider both rationales together – minimizing errors and creating optimal incentives – can we understand why common law jurisdictions have adopted PLR to decide certain types of factual uncertainty and PER to resolve others.

The remainder of the article is therefore structured as follows: Part II explores the incentives created by applying either PER or PLR to uncertainty related to different elements of liability. Following current literature, the first section of Part II shows that in a unilateral accident PLR is preferable to PER when applied to facts for which courts and injurers hold symmetric information (as is usually the case with factual causation and calculation of damages). The second section develops a model that allows us to consider the application of PLR to the determination of negligence. It shows that PER may be superior when the injurer is better informed and cannot reliably convey this

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571 (Ky.1968); Cornfeldt v. Tongen, 295 N.W.2d 638 (Minn.1980); Clayton v. Thompson, 475 So.2d 439 (Miss.1985); Pillsbury-Flood v. Portsmouth Hospital, 128 N.H. 299, 512 A.2d 1126 (1986); Cooper v. Sisters of Charity of Cincinnati, 27 Ohio St.2d 242, 272 N.E.2d 97 (1971); Sherer v. James, 290 S.C. 404, 351 S.E.2d 148 (1986).

9 See Herskovits v. Group Health, 99 Wn.2d 609, 609 (Washington State Courts 1983); Scafidi v. Seiler, 119 N.J. 93 (New Jersey State Courts 1990); Holton v. Memorial Hosp., 679 N.E.2d 1202 (Illinois State Courts 1997). Canadian and English courts have both rejected the “Loss of Chance” doctrine. See, Cottrelle v. Gerard (2003) 233 D.L.R. (4th) 45; Gregg v. Scott [2005] UKHL 2. Courts have adopted other PLR decision rules, such as “market share,” in other circumstances in which factual causation is inherently difficult to prove, such as “indeterminate injurer” (i.e., uncertainty about the identity of the tortfeasor out of a group of potential tortfeasors). See Sindell v. Abbott Laboratories, 607 P. 2d 924 (Cal. Sup. 1980); RESTATEMENT OF THE LAW THIRD, TORTS, LIABILITY FOR PHYSICAL AND EMOTIONAL HARM, § 28(b) cmt. f-j (2010). Finally, PLR was also adopted in circumstances of “indeterminate victim” (i.e., where it is impossible to identify the specific victim out of a group of potential victims). See In Re Agent Orange Product Liability Litigation, 611 F. Supp. 1221 (E.D.N.Y. 1985)

10 I could not find a single case where courts applied PLR and placed partial liability on the defendant according to the probability that she was negligent. In Ybarra v. Spangard 154 P. 2d 687 (1944), the court placed joint and severed liability on 8 doctors and nurses who participated in a surgery, although only one of them negligently caused the harm to the defendant. However, the court in the reasoning for its decision noted the “code of silence” in which all the defendants were complicit, stating that that conduct prevented the plaintiff from finding the real culprit.
information to the court (as is usually the case with negligence). Part III discusses the effect of the different decision rules on the amount and magnitude of courts’ errors, suggesting that when uncertainty is about a binary fact (e.g., causation or negligence) applying PER minimizes both, but when the fact is continuous (e.g., the magnitude of harm) PLR is preferable. The meaning of binary and continuous is further explored as it relates to mass torts. Part IV turns back to legal doctrine to show that courts’ adoption of PLR with regard to uncertainty about the magnitude of harm and certain cases of uncertainty about factual causation is efficient, considering the interaction between the two underlying rationales.

2. Incentives under the Two Decision Rules

Law and economics scholars have examined the effects of uncertainty about each factual requirement. Doing so, several scholars suggested adopting PLR in cases of uncertainty about causation and the magnitude of harm. However, when addressing uncertainty about whether or not the injurer was negligent, scholars have mostly (implicitly) assumed that courts apply PER. Accordingly, this Part first reviews the above literature and then offers a model for analyzing the application of PLR to factual uncertainty regarding whether the injurer’s act was negligent.

2.1 The Use of PLR to Evaluate Harm and Causation.

Creating optimal incentives to take care requires that the expected damages amount, payable by injurers, be equal to the expected harm (Posner 2011, sec. 6.10; Cooter and Ulen 2012, 253–257). Moreover, while super-compensatory or slightly under-compensatory damages awards will not create inefficient deterrence if courts do not make factual errors when assigning liability, if courts do err, over-compensatory damages will result in over-deterrence (Shavell 2007; Dari-Mattiacci 2004; Cooter and Ulen 2012, 220–222). Accordingly, because courts are prone to making errors regarding factual determinations, efficiency demands that expected damages equal expected harm.

This argument justifies adopting PLR whenever courts are uncertain about the magnitude of harm (Shavell 1987, 131–132). Moreover, as further suggested by Kaplow and Shavell (1996), if damages are equal to the average harm, deterrence is optimal and investing in accuracy (i.e., error minimization) is wasteful. Thus applying PLR is superior to PER – the court should calculate the amount of every possible level of harm, multiplied by its respective probability. This calculation leads to the expected harm, thereby creating optimal deterrence. Porat and Stein (2011) showed that the same argument applies for harms that have not yet materialized, but might materialize at
some point in the future.

The next example illustrates the argument:

Example 1: If an injurer causes an accident, the harm that it will cause might be equal to 80, 90, 100, 110 or 120, each with a probability of 20%. After the accident occurs, the court observes evidence regarding the level of harm. However, the information is incomplete, so some level of uncertainty remains. Particularly assume that the court can eliminate three of the possibilities and remain with two possible levels of harm, each of them equally likely.

According to Kaplow and Shavell’s argument, assuming the injurer cannot tell the actual level of harm in advance, courts should set damages to equal 100 in all cases (the ex-ante expected harm) and ignore the evidence about the level of harm in the particular case, saving the cost of collecting and presenting evidence. Moreover, even if the court insists on considering case-specific evidence, it should still apply PLR. To see why, consider, for example, circumstances in which the court knows the magnitude is either 90 or 100, with equal probability. Applying PER, the court should set damages at 90, because the plaintiff can only prove this amount of harm with more than 50% probability (Cohen 2013). Thus, PER leads to under-compensatory damages, which might lead to under-deterrence. If, however, the court applied PLR, it would set damages at 95 \( \left( \frac{90}{2} + \frac{100}{2} \right) \), which is the optimal level.

The argument for using PLR when assessing factual causation follows similar lines to the one above. The debate over the proper way to analyze causation under uncertainty has been extensive both in case law and in legal scholarship. To fully understand the legal attitude towards PLR, we need to first distinguish between three types of risk that may create uncertainty about factual causation. The first type of risk is that created by sources external to the injurer. For example, regardless of the driver’s actions, a passenger may be harmed by an airbag spontaneously inflating due to mechanical malfunction. I will refer to this type of risk as "external risks." The second type of risk is such as would have been created by the injurer, even if she had acted reasonably. For example, if the negligent driver hit a pedestrian, but the accident would have occurred even if the driver took reasonable care, then the risk of hitting the pedestrian was caused by the driver’s actions, but not due to the driver’s negligence. I will refer to this type of risk as "reasonable risks." Finally, the third type of risk is that created by the negligent act of the injurer. For example, if the negligent driver hit a pedestrian, and the accident would not have occurred if the driver had taken reasonable care, then the risk
of hitting the pedestrian was caused by the driver's negligent act. I will refer to this type of risk as "unreasonable risks."

Following the above terminology, we can conclude that, in order to achieve efficient deterrence, the court should only assign legal liability for harm caused by unreasonable risk.\(^1\) In practice, courts often cannot determine which type of risk caused the harm. Moreover, they also treat uncertainty about whether the harm was created by external risk (or by unreasonable risk) differently than they treat uncertainty about reasonable risk.

We will first examine external risks. The next example illustrates a situation in which external risks create uncertainty with regard to factual causation.

\textit{Example 2: A patient goes to a doctor for an examination. At the time of the examination, the patient suffers from a disease which the doctor negligently misdiagnoses. When the disease is finally diagnosed the patient has only a 20\% chance of survival. Ultimately the patient dies from her illness. Had the doctor diagnosed the patient correctly, her chances of recovering from the disease would have been 50\%.}

Based on the facts of the hypothetical, there is a 37.5\% chance that but for the doctor's negligence the patient would have survived, and a 62.5\% chance that the patient would have died regardless of the negligence.\(^2\)

From an efficiency standpoint, PLR creates as good incentives as PER, and sometimes superior, when applied to "external risk" cases. When the court applies PLR, the injurer pays for the expected harm that was caused by her behavior. Rosenberg (1984) showed that in mass-tort cases the use of PLR makes the polluting firm liable only for the harm that it caused, thus creating perfect incentives. Porat (2011) argued that since small added risks, such as in Example 2 above, are taken into account when setting the standard of care, a consistent application of tort law must place liability on the defendant when these risks materialize. Shavell (1985) and Makdisi (1988) went even further, arguing that PLR should be the default rule when dealing with uncertain

\(^1\) Strict liability would also be efficient. However, this article only examines incentives under a negligence regime. See, e.g., (Shavell 1980).

\(^2\) The calculation is straightforward. Since the patient did not survive, we can disregard that probability when trying to assess the cause of death. Since the doctor reduced the probability that she would survive by 30 percentage points, the likelihood that the doctor's negligence caused the patient's death is 30 out of 80, or 37.5\%.
causation. The reasoning is the same in this case as it is for uncertain harm – when applying PLR, the injurer’s expected damages are equal to the expected harm.

We turn now to examining uncertainty caused by reasonable risks. The next example illustrates a situation in which reasonable risks create uncertainty with regard to factual causation.

Example 3: A train passes daily next to a wooden mill. The train emits sparks from the engine. Sparks might ignite and set the mill on fire. The risk to the mill is in direct relation to the speed of the train as it passes by – the faster the train, the more risk it creates to the mill. Considering the risk and cost of care, the reasonable speed is 50 MPH. When the train moves at 50 MPH it still emits some sparks, but the cost of slowing down the train is greater than the reduction in expected harm to the mill. The train passes the mill at 55 MPH, emits a spark, and the mill catches fire.13

Example 3 presents a typical case of uncertain causation due to reasonable risk. The railroad company should be liable only if the spark that ignited the mill would not have been emitted if the train passed the mill at 50 MPH. Since the court cannot distinguish between sparks that are emitted from a slow moving train and the additional sparks that were emitted because of the extra 5 MPH, causation cannot be determined with certainty.

In these types of circumstances, courts have three possible alternatives. First, they may assign full liability, since the negligent act increased the risk of fire. Second, they can adopt a PER rule, assigning full liability only when the negligent act increased the risk by at least twice as much (meaning, when the chances that the fire was caused by the negligence are at least 50%). Third, courts might apply a PLR rule, assigning (partial) liability based on the probability that the negligence caused the fire.

In practice, and unlike courts’ treatment of external risk, when uncertainty is due to reasonable risk some courts disregard it and award full damages, at least as long as they cannot rule out the possibility that the unreasonable risk caused the harm (i.e., whenever the unreasonable risk increased the probability of harm).14 From an efficiency

13 The example is based on Henderson v. Phila., 144 Pa. 461 (Pa. 1891).
standpoint, the way courts resolve uncertainty about factual causation determines the structure of the cost function to the injurer. Standard accident models usually assume that negligent injurers compensate victims for all harm created by the accident, implicitly assuming liability for reasonable risks (Brown 1973; Shavell 2004, 178–182). The injurer’s cost function, under this model, has a discontinuity at the point of optimal investment in care. Examination of this model shows that this discontinuity makes it vulnerable to errors in determining whether the injurer was negligent (Shavell 1987; Kaplow and Shavell 1994; Craswell and Calfee 1986; Cooter and Porat 2014, 17–31).

Kahant (1989) and Dari-Mattiacci (2004) argued that if courts do not award compensation for reasonable risks, the injurer’s cost function is continuous, and the negligence regime is not vulnerable to errors in the determination of liability. However, they assume that the court can determine causation with certainty. Assuming the court cannot tell whether the harm was caused by reasonable risk or unreasonable risk, the result would depend on the decision rule the court adopts.

By applying PER, the court assigns full liability only if there is a more than 50% chance that the accident occurred as a result of unreasonable risk. In Example 3, assume that at 50 MPH the train emits 10 sparks per hour. If the train emits 20 sparks per hour when it drives at a speed of 70 MPH, the court should find the train company liable only when it drives at that speed, and then award full damages. This rule creates bad incentives – the train company knows that it will never be liable for harm when the train moves at a speed lower than 70 MPH, and will therefore drive at an inefficient excessive speed. Applying PER is essentially equivalent to raising the standard of care and ignoring the factual causation requirement when it is applied to reasonable risks. The injurer’s cost function would still have a discontinuity, but the effective standard would be set at an inefficient level.

Stremitzer and Tabbach (2014) showed that applying PLR to uncertainty about reasonable risk is better than any other rule, effectively eliminating the problem of errors in the determination of negligence.

Since courts do not apply PLR to these cases, I assume that courts assign full liability and Tabbach 2014). Recently, Weirib (2015) argued that, following corrective justice, whenever courts are uncertain about the “tortiousness” of the harm, placing full liability is appropriate even if the defendant cannot prove factual causation. Accordingly, to the extent that courts follow corrective justice, this can explain their decision.
even in the presence of reasonable risks, and examine the effects of uncertainty with regard to the negligent conduct of the injurer. The following section explores, for the first time, the use of PLR in the determination of negligence.

2.2 The Use of PLR to Evaluate Negligence

Uncertainty regarding the determination of negligence has two known effects on the incentives of the injurer: first, because the court might err when determining whether or not the injurer was negligent, the injurer knows that by increasing the precaution level she lowers the probability that the court will find her negligent and place full liability on her. Everything else being equal, this effect causes the injurer to overinvest in care. Second, the injurer knows that for every precaution level she might escape liability, and as a result she only partially internalizes the reduction in expected accident costs. Everything else being equal, this effect causes the injurer to underinvest in care. (Miceli 1997; Shavell 1987; Dari-Mattiacci 2004; Craswell and Calfee 1986; Cooter and Ulen 2012; Jason Scott Johnston 1987). Current models assume that the court might make errors, that injurers are aware of that possibility and know the distribution of court errors, and that courts are unaware of it and assume that their determination of facts is accurate. Thus, current models have not implemented PER or PLR.

This section of the article offers a new model of uncertainty regarding the determination of negligence. By assuming that the courts, as well as injurers, know the distribution of errors, the model allows us to examine the effects of uncertainty when applying PER and PLR.

Setting the Model

Consider the risk of accidents occurring between two rational, risk-neutral actors: victim and injurer. Moreover, assume that only the injurer can invest in precaution to reduce the probability of an accident.

Let x be the injurer’s level of care. An accident occurs with a probability of p(x) and causes harm h; p(x) is assumed to be twice continuously differentiable, where p’(x) < 0 and p”(x) > 0.

The social cost of accidents is the sum of the injurer’s investment in precaution and the victim’s expected harm due to accidents, which equals x + p(x)h.

The socially optimal level of x, denoted as x*, minimizes the social costs of accidents. Solving for the first-order condition, we obtain p’(x*)h = −1. Since we assumed that p”(x) > 0 for all x, the second-order condition yields p”(x*)h > 0, proving that x* is a
local minimum.

The optimal level of precaution, \( x^* \), is set as the standard of care under a negligence regime. We assume that once an accident occurs, and negligence is established, the injurer must fully compensate the victim, even if the accident was the result of a reasonable risk. Furthermore, we assume that damages are fully compensatory.

**Proposition 1.** Under a negligence rule, if courts do not err in assessing the injurer's behavior, the injurer will take optimal precautions.

**Proof.** Under these assumptions, the injurer’s expected cost, as a function of level of care, is:

\[
C(x) = \begin{cases} 
  x + p(x)h & \text{if } x < x^* \\
  x & \text{if } x \geq x^*
\end{cases}
\]

The injurer minimizes the second term by investing \( x^* \) in care. By definition, the first term is also minimized by \( x^* \).

Now, assume that courts cannot view the level of care directly, but assess the level of care based on evidence. Specifically, assume that courts observe the level of care \( x \) with some noise or error. Let \( \epsilon \) denote the error in the assessment. We call the precaution observed by courts \( \overline{x} \), where \( \overline{x} = x + \epsilon \).

The probability density of \( \epsilon \), \( f(\epsilon) \), is known to the injurer and to the court.\(^{15} \) \( F(\epsilon) \) denotes the cumulative distribution function of \( \epsilon \). We assume that \( \epsilon \) is independent from the level of care \( x \).\(^{16} \) In other words, when the court observes \( \overline{x} \) it believes that it comes from the same distribution function \( f \) regardless of the size of \( \overline{x} \).

Under these assumptions, from the perspective of the court, the probability that the injurer took less precaution than due care is given by \( (x^* - \overline{x}) \). To see why, note that

\[
P(x \leq x^*|\overline{x}) = P(\overline{x} + \epsilon \leq x^*) = P(\epsilon \leq x^* - \overline{x}) = F(x^* - \overline{x}).
\]

From the injurer's perspective, if she takes precaution level \( x \) then the probability that the court will observe a specific \( \overline{x} \) is given by \( f(\overline{x} - x) \).

\(^{15}\)This assumption is not obvious. The injurer knows the actual level of care, and observes the court’s decision. It could be argued that the injurer is in a better position to know the court’s error function than the court. However, that requires the injurer to observe several decisions, or to observe the actual level of precaution other injurers took. Assuming injurers are not repeat players, it is likely that injurers will have the same information as the courts regarding the distribution of errors.

\(^{16}\)This is a common assumption in the literature. See, e.g. (Shavell 1987, 93–99).
**Proposition 2.** If the court knows the distribution of $\varepsilon$, then there will be no bias in determining liability, regardless of whether the distribution is biased.

**Proof.** The distribution function of the error is biased if the error causes the court to systematically underestimate or overestimate the level of precaution taken by the injurer. However, since the probability density function $f(\varepsilon)$ is known to the court, it knows if it is biased, and can correct its estimation, thus, $\int_{-\infty}^{\infty} tf(t)dt = 0$ (or the average of $\varepsilon$ is 0).

Courts might still systematically err if the distribution of the error is skewed (i.e., the median error is higher or lower than zero). However, under a rule examining the probability of the deviation of the observed behavior from the standard of due care (e.g., PER or PLR), courts would consider this possibility before assessing the probability that the injurer was negligent.

To conclude, when the court knows the distribution of the errors and assesses the probability of fault based on that distribution, it might err in particular instances, but will be correct on average. Accordingly, because biased functions will not cause biased decisions, we can assume for simplicity that $f$ is a symmetric function, with mean and median equal to zero. Since the median error is zero, it follows that $F(0) = 0.5$. Following this structure, we can now compare the efficiency of the different decision rules.

**Incentives under PER**

Under PER, the court imposes full liability if and only if the probability that the injurer was negligent is greater than 50%. Recall that from the court's perspective, the probability that the injurer was negligent is $F(x^* - \overline{x})$. Thus, under PER the court will find the injurer negligent whenever $F(x^* - \overline{x}) \geq 0.5$.

**Proposition 3.** Under PER, the court should impose liability whenever the observed level of precaution is lower than the standard of care ($\overline{x} < x^*$).

**Proof.** Recall that from the court's perspective, the probability that the injurer took less precaution than the due care standard is given by $F(x^* - \overline{x})$. Since $F(0) = 0.5$, and $F$ is monotonically increasing, the court should impose liability whenever $\overline{x} < x^*$.

**Proposition 4.** Under a negligence rule, when the court uses PER in order to assign liability under uncertainty, the injurer may take under- or over-precaution.
Proof. From the injurer's perspective, the probability that a particular $x$ will be realized is $f(\bar{x} - x)$ for any $x$. Since courts find the injurer liable whenever $\bar{x} < x^*$, the probability that the court will find the injurer liable under PER is given by:

$$\int_{-\infty}^{x^*} f(\bar{x} - x) \, d\bar{x} = F(x^* - x).$$

It follows that under the PER, the injurer's cost function is:

$$C_{PER} = x + F(x^* - x)P(x)h$$

Differentiating Exp. 2, we obtain:

$$C'_{PER} = 1 - f(x^* - x)P(x)h - F(x^* - x)P'(x)h$$

Exp. 3 has three components. The first, 1, is the marginal cost of care. The second is the marginal reduction in expected liability, since every investment in precaution reduces the probability that the court will find the injurer negligent. All else being equal, the effect of uncertainty here is to induce the injurer to take more care. The third component is the reduction in accident costs due to the added level of care, multiplied by the probability that the court will impose liability. Because the injurer might escape liability even if she invested in care less than the due care standard, here the effect contrarily is to cause the injurer to take less care. We will refer to the first effect of uncertainty as the "private benefit of added precaution," since it describes a private, and not public, benefit to the injurer created from each additional unit of precaution. We will refer to the second effect as the "partial internalization of added precaution," since the injurer internalizes only part of the reduction in the social costs of accidents.

Evaluating Exp. 3 at $x = x^*$, where $p'(x^*)h = -1$, we obtain:

$$C'_{PER}(x^*) = 1 - f(0)p(x^*)h - F(0) = \frac{1}{2} - f(0)p(x^*)h$$

This result is similar to the results commonly presented in the literature (Miceli 1997; Shavell 1987; Dari-Mattiacci 2004; Craswell and Calfee 1986; Cooter and Ulen 2012). However, while previous models have (implicitly) assumed that the courts are unaware of the possibility of an error, but believe that their decision is accurate, the above analysis shows that the result holds, even if courts do not hold this belief and as long as they apply PER to cases of uncertainty. Moreover, while previous articles have discussed the effects of biased errors by the court (Dari-Mattiacci 2004; Craswell and Calfee 1986;
Jason Scott Johnston 1987), the current model shows that when the courts know the distribution function of errors, then the errors themselves should not be biased.

**Proposition 5.** Evaluating the level of uncertainty by comparing it to the optimal expected harm allows us to know whether the uncertainty regarding the behavior of the injurer causes over- or under-deterrence under PER.

**Proof.** According to Exp. 4, the question whether the injurer will underinvest or overinvest in precaution is determined by two parameters: the optimal expected harm and the marginal reduction in the probability that the injurer will be found negligent. In order to offer some insights, I assume that the error is normally distributed.\(^{17}\)

Following the previous assumptions, the median and the mean of the distribution equal zero, so \(\varepsilon \sim N(0, \sigma)\). Since we have seen that the two parameters that affect the injurer’s behavior are the probability density function and the optimal expected harm, we can express the connection between them by setting the size of the standard deviation as a multiplier of the optimal level of expected harm, or \(\varepsilon \sim N(0, \sigma \ast p(x^*)h)\).

Evaluating Exp. 4, while assuming normal distribution of \(\varepsilon\), we obtain:

\[
(5) \ C'_{PER}(x^*) = \frac{1}{2} - \frac{1}{\sqrt{2\pi}\sigma p(x^*)h}p(x^*)h = \frac{\sigma - \sqrt{\frac{1}{\pi}}}{2\sigma}
\]

The meaning of the result is straightforward. Since \(\sigma\) is a multiplier of the optimal expected harm, we can see that whenever the standard deviation is smaller than approximately 80\% (\(\sqrt{\frac{2}{\pi}}\)) of the optimal expected harm, PER will result in over-deterrence. However, if the standard deviation is larger, PER will cause under-deterrence and the injurer will underinvest in precaution. Notice that at the limit, the expression yields \(\lim_{\sigma \to \infty} \frac{\sigma - \sqrt{\frac{1}{\pi}}}{2\sigma} = \frac{1}{2}\), which means that as the uncertainty gets larger, the effect of partial internalization of added precaution stays the same, but the private benefit from added precaution gets smaller (it approaches zero as the standard

\(^{17}\) I can show qualitatively similar results for uniform and triangular symmetrical distributions. I use here normal distribution because it is the most realistic option. Since the court’s decision is made up of a large number of small decisions (e.g., the meaning of the dispersion of glass at the scene of the accident, the skid marks on the road, weather conditions at the time of the accident, etc.), all of which are prone to error, it is fair to assume normal distribution, according to the central limit theorem.
deviation multiplier approaches infinity).

Following the analysis thus far, it should be obvious that the problem of over-deterrence is not necessarily more common than the problem of under-deterrence. Indeed, when optimal expected harm is relatively small, compared to the investment in precaution, under-deterrence seems more likely to occur than over-deterrence.\(^{18}\)

**Incentives under PLR**

Consider now the alternative decision rule, under which the court imposes partial liability according to the probability that the injurer took less than due care. As mentioned above, the probability that the court will observe any particular \(\bar{x}\), given a particular investment in \(x\), is given by \(f(\bar{x} - x)\).

*Proposition 6.* When uncertainty causes over-deterrence under PER, a shift to a regime of PLR can increase or decrease the problems created by uncertainty.

*Proof.* The following function describes the cost to the injurer under PLR.

\[
(6) C_{PLR} = x + P(x)h \int_{-\infty}^{\infty} f(\bar{x} - x)F(x^\star - \bar{x}) d\bar{x} \quad^{19}\]

Differentiating Exp. 6 and evaluating the result at \(x^\star\), we get:

\[
(7) C'_{PLR} = 1 - P(x^\star)h \int_{-\infty}^{\infty} f'(\bar{x} - x^\star)F(x^\star - \bar{x})d\bar{x} - \int_{-\infty}^{\infty} f(\bar{x} - x^\star)F(x^\star - \bar{x})d\bar{x} - \frac{1}{2}
\]

From Exp.7, it is easy to see that both effects of uncertainty identified under PER remain under PLR. The partial internalization of added precaution effect is even of the same magnitude, meaning that at the optimal level of care the injurer only internalizes half of the marginal reduction of the expected harm. However, the private benefit from added precaution effect is smaller at the optimum level of care. Specifically, if we assume that \(\epsilon\) is normally distributed, then the second term is equal to \(-\frac{1}{2\sqrt{\pi}\sigma}\), and the entire function yields \(\frac{\sqrt{\pi} \sigma - 1}{2\sqrt{\pi} \sigma}\).

This outcome is rather intuitive. Because the injurer knows she will pay some damages even when the court finds her negligent with a probability below 50% (a case for which

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\(^{18}\) Some scholars have argued that the problem of over-deterrence is a lot more common (Shavell 2002; Shavell 2004; Bar-gill 2001; Cooter and Ulen 2012, 220–222).

\(^{19}\) Note that \(\bar{x}\) is used here as a parameter and not a variable.
she would not have paid damages under PER), but the amount of damages will be smaller than the amount under PER if the probability is above the 50% threshold, the private benefit that the injurer derives from added precaution is smaller at the optimum than under PER.

Fig. 1 shows the value of the first-order solution, assuming an optimal level of precaution.

**Fig. 1: The effect of PER and PLR on the Incentive to Invest in Care**

Fig. 1 shows that the change from over-deterrence to under-deterrence occurs under PER when the standard deviation is approximately 80% of the optimal expected harm, while under PLR it occurs when the standard deviation is less than 60% of optimal expected harm. This, however, does not mean that the injurer will always take a lower level of precaution under PLR. When the size of the uncertainty (σ) is very small relative
to the optimal expected harm, the injurer can avoid liability almost completely under PER by investing slightly more in precaution. This effect only occurs at very low levels of uncertainty (up to 25% of the expected optimal damages). Fig. 2 and Fig. 3 show the cost function to the injurer, given PER or PLR. In Fig. 2 the standard deviation equals 10% of the optimal expected damages, and in Fig. 3 it is set at 30%. In these examples both decision rules lead the injurer to take excessive care. However, when the size of $\sigma$ is relatively small the inefficiency is greater for PLR.

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20 The general result holds for other forms of distribution, although at different levels of standard deviation.
Other than situations of small standard deviations, PLR induces the injurer to take less care than under PER. This means that when the uncertainty causes over-deterrence PLR can create either better incentives to take care than PER, or worse. However, when the uncertainty causes the injurer to take too little precaution, PER is generally superior. Using computer simulation, Fig. 4 shows the level of care that minimizes the cost function to the injurer, as a function of the level of uncertainty ($\sigma$).\textsuperscript{21}

\textsuperscript{21} The simulation was performed using Wolfram Mathematica 9. The error's PDF is a normal distribution function. To check the robustness of the results, I ran the simulation using different $p(x)$ functions. The results did not substantially change as long as the standard deviation was expressed as a multiplier of the expected optimal harm. The result also holds for uniform and triangular distributions of the error.
Fig. 4: Investment in precaution under PER and PLR as a function of $\sigma$

The x-axis in Fig. 4 represents the size of standard deviation $\sigma$, as a fraction of the optimal expected harm. The y-axis represents the possible levels of precaution ($x$). The red and blue curves show the level of $x$ that the injurer takes under PER and PLR, respectively. It makes sense to separate the results into three sections. The first section, at the left end, shows small levels of uncertainty. Here PER induces the injurer to take less care than PLR. Since both rules create over-deterrence, PER is superior. The section in the middle shows intermediate levels of uncertainty. Here, both rules over-deter the injurer, but PLR creates less over-deterrence and is superior. In the last section, both rules induce the injurer to take less care than the optimum. Since PLR makes the injurer take less care than PER, it is less efficient.

**Solving Over-Deterrence Caused by Uncertainty under PER**

PER creates better incentives than PLR for low and high levels of uncertainty. However, in cases of intermediate levels of uncertainty PLR is supposedly superior. If it were possible to identify categories of cases where the typical level of uncertainty is known, we could have suggested that the court adopt one rule or the other. However, it seems unlikely that we can identify a typical level of uncertainty for a category of cases. Under these conditions, *ex-ante* incentives offer us little help in identifying a superior decision rule.

If, however, we can find a solution for over-deterrence under PER, then we could always use PER, and not PLR. The court needs less information to apply PER (it only needs to establish whether the probability crossed the 50% threshold, and not the exact...
probability), so if we find a solution for over-deterrence under both PER and PLR, applying it only to PER might be more efficient.

Such a solution exists, at least theoretically – the court can set under-compensatory damages to correct for the over-deterrence created by uncertainty.\footnote{If damages are under-compensatory this might have positive effects on the victims’ incentives. Assuming victims can take unobservable precaution, the fact that they might get compensation even if the injurer was not negligent induces them to invest too little in care. Partial compensation is then better than full compensation in that it makes the victims internalize more of the accident costs.} Furthermore, if not all victims sue for damages, under-enforcement will have the same effects as under-compensation. Appendix 1 shows how applying partial damages to the model can solve the problem of over-deterrence.

In practice, however, courts are usually unable to intentionally use partial compensation in order to offset the problem created by uncertainty. Tabbach and D’Antoni (2014) showed that limiting damages might have unexpected and unwanted results when it is socially undesirable that the victim take any amount of care, although she is able to do so. In these circumstances, partial compensation might induce victims to take too much care, and injurers to take too little.

If, however, under-enforcement is a natural phenomenon, and not a decision made by the court, it might have the same effect on deterrence. Indeed, in some types of accidents it is well established that a large percentage of the victims does not sue. For example, in medical malpractice cases some researchers showed that as low as 5% of the patients who suffered harm due to medical malpractice file a suit (Baker 2005; Studdert et al. 2006). In other cases, such as pollution, the latent effect of exposure may cause very few victims to be fully compensated. If under-enforcement is an exogenous effect that cannot be controlled by the court, we should consider it as another factor, other than uncertainty, that should be taken into account. The analysis in Appendix 1 shows that if uncertainty and under-enforcement cause under-deterrence, the court can always create optimal incentives by awarding punitive damages in cases of gross negligence.

2.3 Concluding remarks on the efficiency of PLR

The analysis in this part has shown that although PLR is at least as efficient as PER when applied to the calculation of damages and to determining causation, the outcome is ambiguous when applied to the question of negligence. When the court determines negligence, PLR might be either superior or inferior to PER, depending on the magnitude
of the uncertainty. However, in cases where PLR is superior, the court can implement the use of under-enforcement, combined with punitive damages in cases of gross negligence, to create optimal incentives. The use of punitive damages in these cases might be better than applying PLR, since very few cases of gross negligence would actually appear before the court, so it would only rarely have to apply the rule.

There is a common element that might explain the difference between the effects of PLR on the determination of negligence and its effects on the determination of causation and the magnitude of harm. Notice that the injurer, by definition, has full information regarding the actual level of precaution she took. Thus, when the injurer decides how much to invest in care she can take into account the expected error, which means that the distribution of errors might affect the injurer’s behavior even if the court is correct on average.

However, the injurer faces the same uncertainty as the court with regard to the existence of causation and the magnitude of harm. In these cases, as long as the expected damages equal the expected harm, the injurer is unaffected by errors or the shape of their distribution function. The result is only true if we look at a unilateral accident model. If the victim can invest in precaution, and knows the actual level of harm she might suffer, then again the decision rule would affect the victim’s incentives.

3. Error Minimization

Legal scholars often assume that the goal of evidentiary law is to minimize the costs of errors and the costs of error minimization efforts (Posner 1999; Stein 2005). Rules of procedure and evidence, under this view, should incentivize the parties to invest in collecting and presenting evidence only as long as the cost of collecting the evidence is lower than the reduction in the expected costs of errors. Nevertheless, even if parties optimally invested in collecting and presenting evidence and only presented evidence with probative value, courts would still face some degree of uncertainty regarding every factual decision. Decision rules are then used to divide the risk of errors between the parties.

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23 We can think of making a mistake by the court as an accident, and investing in evidence as a precaution. According to this view, the goal of evidence law is similar to the goals of tort law – to reduce the sum of the costs of accident and the costs of care.
3.1 Reducing errors in binary variables

All else being equal, PER reduces the (absolute) number of errors when applied to a binary variable (Kaye 1982; Levmore 1990; Stein 2005). To understand why, first consider the fact that for binary variables, if the probability of outcome ‘A’ is \( p \) then the probability of the alternative outcome ‘B’ is \( (1 - p) \). Accordingly, courts facing factual uncertainty that wish to minimize errors should determine that the fact is ‘A’ whenever \( p > (1 - p) \) (i.e., \( p > 0.5 \)). Finally, when the costs associated with false positive are different than those associated with false negative, reducing the cost of errors might justify adopting different thresholds (e.g., ‘beyond reasonable doubt’). However, this is unlikely to be the case in civil disputes (Stein 2005).

PLR seems especially problematic with respect to the frequency of errors, regardless of their magnitude. This is because applying PLR to a binary factual variable means that courts make an error in each of their decisions. Moreover, even if minimizing errors is understood in terms of minimizing the (aggregated) amount of compensation wrongly allocated, PLR creates more errors than PER in almost every type of circumstances (Kaye 1982; Levmore 1990). To illustrate this idea, assume that when outcome ‘A’ occurs the injurer should be found liable for harm in the magnitude of 1. If there is a probability \( p \) that ‘A’ occurred, then when applying PLR the court will award the plaintiff damages in the sum of \( p \). Applying PLR to these type of circumstances means that with probability \( p \) the victim is undercompensated in the amount of \( (1 - p) \), and with probability \( (1 - p) \) the victim is overcompensated in the amount of \( p \). Thus, the aggregated amount of misallocation of damages is \( 2p(1 - p) \). Under PER, however, when \( p > 0.5 \) the amount of misallocation of damages is \( (1 - p) \). Comparing the two rules, we see that \( 2p(1 - p) > (1 - p) \), for any \( p > 0.5 \).

3.2 Reducing Errors in Continuous Variables

When the uncertain variable is continuous, the frequency of errors is meaningless since every decision is wrong (because the probability of any particular result is zero). Accordingly, we can only consider the amount of misallocated compensation.

Let ‘\( c \)’ denote an uncertain continuous variable, distributed between ‘\( a \)’ and ‘\( b \)’, where \( f(c) \) is the probability density function of ‘\( c \)’ and \( F(c) \) is the cumulative distribution function. Thus, when the court’s determination of ‘\( c \)’ is set at ‘\( \hat{c} \)’, the sum of possible errors is given by:

\[
(12) \text{Error}(\hat{c}) = \int_a^\hat{c} f(c)(\hat{c} - c) \, dc + \int_{\hat{c}}^b f(c)(c - \hat{c}) \, dc
\]
It is easy to see that Exp. 12 is minimized when the court sets \( \hat{c} \) to the weighted mean of the continuous variable \( c \), so, \( \hat{c} = \int_a^b f(c) \cdot c \, dc \). Applying PLR, courts would choose the weighted mean, since it represents the sum of all possible outcomes multiplied by their respective probabilities.

It is impossible to apply PER to a continuous variable directly, since there is no single outcome that has a probability greater than 50%. However, PER can be applied to the cumulative distribution function. Following this approach, courts would set \( \hat{c} \) at the median point (i.e., when the probability that \( 'c' \) is greater (or smaller) than \( '\hat{c}' \) is 50%, \( F(\hat{c}) = 0.5 \)) (Cohen 2013). This would equate the frequency of courts' overassessment of \( c \) (\( \hat{c} > c \)) to the frequency of underassessment (\( \hat{c} > c \)), but because the magnitude of errors may be different, it would usually mean that the median (and therefore PER) is not error-minimizing.\(^{24}\)

4. Reassessing Current Application of PLR

In the previous parts I examined the respective effects of applying PLR and PER on the \textit{ex-ante} incentives of injurers to take precautions and on \textit{ex-post} error minimization. I can now turn to examining courts' application of PLR and PER to different types of factual uncertainty, in light of these two rationales.

4.1 The Use of PLR in the Calculation of Damages

Courts facing uncertainty regarding the calculation of damages apply PLR in most, if not all cases.\(^{25}\) Calculating damages, especially in personal injury cases, requires courts to make various factual determinations. These include estimation of continuous variables (e.g., the accident's effect on the victim's life-expectancy, future earnings, work-life expectancy, etc.) as well as binary ones (e.g., whether or not a victim suffering from a head injury, who has a 20% probability of suffering from epilepsy in the future, will in fact suffer from it). When the courts use weighted means, rather than medians, to assess the magnitude of harm, they are applying PLR and not PER, because they ascribe weight to probabilities lower than 50%.\(^{26}\)

\(^{24}\) Note that if \( 'c' \) is distributed symmetrically, then the median is equal to the weighted mean, so both PER and PLR reach the same result. In this case, and only in this case, PER is error-minimizing, but it is still not preferable to PLR.

\(^{25}\) Supra note 6.

\(^{26}\) When the court has no information regarding the future earning capacity of the plaintiff (as is the case with children), courts use the weighted average salary, and not the median, to calculate
We saw that using PLR to calculate damages creates better incentives and, at least for continuous variables, also minimizes errors. I suggest, moreover, that PLR would also be error-minimizing when applied to binary variables in determining the magnitude of harm. To understand why, observe that the magnitude of harm in itself is a continuous variable and, accordingly, in order to minimize errors PLR should apply. In other words, because determining the amount of harm requires integrating several variables, applying PLR to the magnitude of harm in general would require courts to apply PLR to every sub-variable as well.

In conclusion, when uncertainty is about the magnitude of the harm, PLR should be applied both from the perspective of error minimization and from the perspective of creating optimal *ex-ante* incentives to take care. This conclusion, it should be noted, is also consistent with courts' extensive adoption of PLR when determining the magnitude of harm in practice.

Moreover, it is only when both rationales are considered together that we can explain courts' insistence on hearing case-specific evidence. Recall that following Kaplow and Shavell (1996), taking case-specific evidence into account does not facilitate the creation of efficient incentives and therefore represents a pure cost. Accordingly, if courts were concerned solely with creating optimal incentives, we should have observed reluctance on their part to consider such evidence in their decisions. In practice, however, courts do base their decisions on case-specific evidence, and spend considerable judicial resources in an attempt to accurately evaluate such evidence in the case at hand. This practice, then, can be understood once we consider the rationale of minimizing errors, for which case-specific evidence is beneficial.

### 4.2 The Use of PLR in the Determination of Factual Causation

Contrary to uncertainty about the magnitude of harm, when courts face uncertainty about factual causation, they tend to apply PER. This preference can be explained in light of the fact that, because causation is binary variable (i.e., either the injurer's unreasonable risk caused the harm or it did not), applying PER will usually be error-minimizing. Moreover, as discussed above, as long as the distribution of errors is symmetrical, PER and PLR both create efficient incentives to take precautions. Thus, in these types of circumstances, integrating the two rationales shows that PER is better
Nevertheless, when the distribution of error is asymmetrical, the application of PER should be reconsidered, and this is indeed what courts do. Thus, we can divide the instances where courts apply PLR to factual causation into three types of circumstances: indeterminate injurer, indeterminate victim, and prior knowledge of external risks.

In cases of indeterminate injurer, several jurisdictions have adopted the doctrine of “market share liability.”27 Under this doctrine, when each victim (out of a group of victims) is unable to prove from which specific manufacturer, out of a group of negligent manufacturers producing identical products, she bought the product, liability is allocated according to the market share of each manufacturer. Each victim receives full compensation and each manufacturer pays the amount of harm it actually created. Thus market-share liability is not only efficient, but also error-minimizing.

In cases of indeterminate victim, courts are unable to determine which victim, out of a group of potential victims suffering harm after being exposed to an unreasonable risk, suffered the harm as a result of the injurer’s negligent conduct (as opposed to harm caused by reasonable or external risk). In several instances, courts have considered the application of PLR to these types of cases.28 Applying PLR to these circumstance creates better incentives for the injurer (Rosenberg 1984), because it holds the injurer liable for its expected harm. Moreover, the application of PLR may also be error-minimizing. Thus, as Levmore (1990) showed, when the injury was caused by a reoccurring wrong, as in the case of mass torts, applying PLR reduces the number of errors from the injurer’s perspective. In other words, when the unreasonable risk created by the injurer may affect many potential victims and harm some of them, asking which harm was created by the negligent behavior is akin to asking how many victims suffered harm as a result of the unreasonable risk. Rearticulating the question this way turns the binary variable into a discrete variable with binomial distribution. Assuming the number of potential victims is \( n \) and that the probability that the injurer caused the injury to each of them is \( p \), PLR emerges as the error-minimizing decision rule, as can be shown by the following Exp. (13):

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27 Sindell v. Abbott Laboratories, 26 Cal. 3d 588 (1980). RESTATEMENT OF THE LAW THIRD, TORTS, LIABILITY FOR PHYSICAL AND EMOTIONAL HARM, § 28(b) cmt. f-j (2010). It is not a case of indeterminate victims, since all victims were injured as a result of a negligent conduct. 
\[(13) \text{Error}(k) = \sum_{i=0}^{k} i \cdot \binom{n}{i} (1 - p)^{n-i} p^i (k - i) + \sum_{i=k+1}^{n} i \cdot \binom{n}{i} (1 - p)^{n-i} p^i (i - k)\]

In Exp. 13 \(k\) represents the court's estimated number of victims that suffered harm as a result of the unreasonable risk. Exp. 13 measures the expected size of court errors, by weighing each possible number of victims (\(i\)) by its probability, and measuring the difference between the number of victims to the court's estimation. It is easy to see that setting the court's estimation to the average number of victims (\(k = np\)), minimizes the number of expected errors in Exp. 13. Therefore, in cases of both indeterminate injurer and indeterminate victim, applying PLR creates efficient incentives to take care and is error-minimizing.

The cases where the injurer has prior knowledge of external risks are less clear. Consider again Example 2:

**Example 2: A patient goes to a doctor for an examination. At the time of the examination the patient suffers from a disease which the doctor negligently misdiagnoses. When the disease is finally diagnosed the patient has only a 20% chance of survival. Ultimately the patient dies from her illness. Had the doctor diagnosed the patient correctly her chances of recovering from the disease would have been 50%.

Until the 1980s courts routinely rejected cases of medical malpractice where the plaintiff could not prove causation beyond the preponderance of the evidence. This problem is frequent in medical malpractice cases because of high levels of external risk.\(^{29}\) During the last 40 years, however, many jurisdictions have adopted the doctrine of "lost chance of recovery," according to which plaintiffs are entitled to compensation based on the probability that their death was caused by the doctor's negligent behavior.\(^{30}\)


Circumstances of malpractice are also unique, in that doctors often have prior knowledge of the external risk that victims face. If courts were to implement PER then, whenever the external risk was sufficiently high, doctors would know *ex-ante* that they will never be found liable, because victims would never be able to prove causation under a PER rule. Therefore, PLR is paramount with regard to creating efficient incentives for doctors to invest in precautions. However, because in these circumstances uncertainty is about a binary factual variable (i.e., causation), PER is the error-minimizing decision rule. Accordingly, courts’ adoption of PLR can be explained as a normative preference for creating efficient incentives over minimizing errors.\textsuperscript{31}

To conclude, in most cases dealing with uncertainty about causation PER would minimize errors, without distorting the injurer’s incentives to invest in precautions. Accordingly, PER should usually be preferred by courts to resolve this type of uncertainty. However, whenever either PLR is the superior error-minimizing rule or PER distorts injurers’ incentives to take care, PLR is preferred by courts.

4.3 The Use of PLR in the Determination of the Injurer’s Conduct

No jurisdiction has adopted PLR with respect to uncertainty about reasonableness of the injurer’s actions. This can be explained when the two rationales are considered. First, given that determining whether or not the injurer violated the duty of care is binary in nature, PER is clearly the error-minimizing rule.\textsuperscript{32} Second, as the model offered in Part II suggests, PER also creates better incentives than PLR when resolving this type of uncertainty. Accordingly, because PER is superior to PLR in both minimizing errors and creating efficient incentives, it should come as no surprise that courts refuse to adopt PLR when resolving uncertainty about the injurer’s actions.

References


Bar-gill, Oren. 2001. “Does Uncertainty Call for Comparative Negligence?” *The Harvard*

\textsuperscript{31}The courts in England have rejected the ”lost chance of recovery” doctrine. See Gregg v. Scott [2005] UKHL 2. This difference from American law can be explained by a normative preference for *ex-post* error minimization over *ex-ante* efficient incentives.

\textsuperscript{32}The determination of negligence does not have to be binary. Kolber (2014) argued that by reducing the question of negligence to a binary state the court loses ‘morally relevant information.’ According to Kolber’s suggestion, liability should increase as the conduct becomes less reasonable, in order to reflect the immorality of the injurer’s behavior.


Appendix

**Proposition 6.** For every case in which PER causes over-deterrence \((\sigma < 0.8)\), it is possible to correct the incentives of the injurer by allowing partial damages or by limiting enforcement by victims.

**Proof.** Reducing damages awards by a factor, or allowing only some of the victims (chosen randomly) to file a suit, affects the injurer’s incentives in the same way. Formally, it can be stated in the following way:

\[
C_{UE} = x + \alpha F(x^* - x)P(x)h
\]

Where \(\alpha\) denotes the percentage of the victims allowed to place a suit, or the factor by which damages are reduced (for convenience, we will refer to \(\alpha\) as the level of under-enforcement). Differentiating Exp. 8 and evaluating the result at \(x^*\), we can solve for \(\alpha\):

\[
\alpha = \frac{2\sqrt{\pi} \sigma}{\sqrt{2} + \sqrt{\pi} \sigma}
\]

According to Exp.9, the court can always accurately offset the effect of over-deterrence created by uncertainty if it can control the level of under enforcement \(\alpha\). In practice, however, courts are usually unable to intentionally use under-enforcement (by barring plaintiffs from filing suits) or partial compensation in order to offset the problem created by uncertainty. For one thing, we might have noneconomic reasons to want to allow every victim who wishes to, to file a suit. As for using damages that are not fully compensatory, Tabbach and D’Antoni (2014) showed that limiting damages might have unexpected and unwanted results when it is socially undesirable that the victim take any amount of care, although she is able to do so. In these circumstances, partial compensation might induce victims to take too much care, and injurers to take too little.

Another way to think of \(\alpha\) is as a natural phenomenon, and not a decision made by the court. If under-enforcement is an exogenous effect that cannot be controlled by the court, we should consider it another factor, other than uncertainty, that should be taken into account.

33 UE stands for "Under Enforcement."
34 Exp. 9 is not limited to cases of under-enforcement. Mathematically, \(\alpha\) can be larger than 1. In cases where uncertainty causes under-deterrence, over-enforcement can solve the problem the same way under-enforcement solves the problem of over-deterrence. This means that in cases of high levels of uncertainty regarding negligence, the courts should use extra-compensatory (or punitive) damages to offset the effects of uncertainty.
Fig. 5 shows the connection between the level of under-enforcement and the level of uncertainty. The black line represents the optimal level of under-enforcement for different sizes of uncertainty (presented in terms of σ, a multiplier of the optimal expected harm). The white area above the curve represents a state of over-deterrence, and the gray area under the curve represents a state of under-deterrence, caused by under-enforcement.

**Proposition 7.** When under-enforcement causes under-deterrence, it is always possible to correct the incentives by awarding punitive damages for gross negligence.

**Proof.** It is well established that punitive damages can offset the effects of under-enforcement. Polinsky and Shavell (1998) have shown how setting damages by multiplying the harm by the multiplicative inverse of the under-enforcement \( \left( \frac{1}{\alpha} \right) \) in our model can completely offset the effect of under-enforcement (assuming the injurer is risk-neutral). However, Polinsky and Shavell’s model assumes that the court does not err when assessing the injurer’s negligence. In our model, we want to only partially offset the effect of under-enforcement, since part of the effect is desirable.

It might still be possible to use some (low) level of punitive damages in every case, in order to achieve optimal deterrence. The court could set the damages multiplier in a way that results in optimal deterrence. However, there are two reasons why awarding
punitive damages in every case is not the best alternative. First, when the court awards punitive damages to every plaintiff, the proportion of the victims that sue after the accident changes. Under Polinsky and Shavell’s model, this is a welcome outcome since under-enforcement only creates costs, but not benefit.\footnote{This is true only if we assume that the costs of litigation are zero. If litigation creates positive costs, it might be preferable to allow only a subset of the plaintiffs to sue, and award them punitive damages, even if courts do not make any errors.} According to the current model, an increase in private enforcement might be undesirable. Second, the calculation of punitive damages becomes a very difficult task when the court needs to take into account not only the level of under-enforcement but also the effects of uncertainty. However, if the court awards punitive damages only for gross negligence then it avoids the need to calculate the correct level of punitive damages in every case. Assuming the legal regime creates optimal incentives to the injurer, only in few cases would the injurer be considered grossly negligent, and punitive damages awarded. Furthermore, in such cases the court would not have to take into account the effects of uncertainty when calculating punitive damages. If the standard of gross negligence was set optimally, the court could use the same damages multiplier offered by Polinsky and Shavell.

Let’s assume that the court sets a second standard of care, at $(x^* - g)$, for grossly negligent behavior. When the level of precaution that the court observes is below the gross negligence standard (formally: $\bar{x} < x^* - g$), then the court awards punitive damages by multiplying the amount of damages by $\frac{1}{\alpha}$. By adjusting Exp. 8, we get:

$$C_{PD} = x + aF(x^* - x)P(x)h + aF(x^* - g - x)P(x)h \frac{1}{\alpha}$$

Differentiating Exp. 8 and evaluating the result at $x^*$, we get:

$$C'_{PD} = 1 - \frac{a}{2} - a f(0)P(x^*)h - F(g) - f(g)P(x^*)h$$

The result is made up of five expressions: the first is the marginal cost of care. The second is the marginal reduction in accident cost, multiplied by the probability of being found negligent (0.5) and by the level of under-enforcement $\alpha$. The third is the marginal reduction in the probability of being found liable, multiplied by the entire optimal harm and the level of under-enforcement. The fourth is the marginal reduction in accident cost, multiplied by the probability of being found grossly negligent. The final expression is the marginal reduction in the probability of being found grossly negligent, multiplied

\footnote{PD stands for a regime of punitive damages.}
by the optimal expected harm.

In order to evaluate Exp. 11 under the assumption that $e$ is normally distributed, we need to express the level of $g$ (the deviation from optimal care that is considered gross negligence) in terms of $\sigma$. As may be recalled, $\sigma$ itself is a multiplier of the optimal level of expected harm. If the standard deviation of the error is $\sigma * p(x^*) h$ and $g$ is a multiplier of the standard deviation, or $g * \sigma * p(x^*) h$, then we can find an optimal level of $g$ for every combination of $\alpha$ and $\sigma$.

Fig. 6 shows the optimal levels of under-enforcement and uncertainty, for different levels of $g$. Remember that $g$ is expressed as a multiplier of the standard deviation. When $g=4$ it is so unlikely to be found grossly negligent that the line is practically identical to the line of the optimal level of under-enforcement in Fig. 5.

The curves drawn in Fig. 6 are just a few examples of different possible levels of $g$. It is easy to see from the graph that for every combination of $\alpha$ and $\sigma$, it is possible to find a

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37 Formally the expression is: $\alpha = \frac{\sqrt{\pi} \cdot \frac{\sigma^2}{\sigma + \sqrt{\pi} e^2} \cdot \text{erfc} \left( \frac{\sigma^2}{2\sqrt{\pi} e^2} \right)}{\sqrt{\pi} e^{\sigma + \sqrt{\pi} e^2}}$
level of $g$ that will create optimal incentives.\footnote{It is also possible to solve it mathematically, by solving the expression in FN 37 for $g$, but since $g$ is under the error function, this is a burdensome task.} Even if the court is limited in some way, and is forced to use only a limited number of optional levels of $g$, then choosing the curve closest to the specific combination of $\sigma$ and $\alpha$ would significantly improve the incentives that the legal rule creates.

A second way to interoperate the second standard (formally $x^* - g$) is to think of it as a different standard of proof, instead of a standard of gross negligence. For example, if $g = 1.645$, then when the level of precaution observed by the court is less than $x^* - 1.645\sigma$, the court knows that the likelihood that the injurer was negligent is 95%. Assuming the legal standard of proof of "beyond a reasonable doubt" is equivalent to proving with a probability of 95% or higher, then essentially when the level of care observed by the court is lower than $(x^* - g)$, it knows that the injurer has acted negligently beyond a reasonable doubt.

This second interpretation offers us a new explanation for the duality of criminal and private enforcement. When the combined effect of under-enforcement and uncertainty is to create insufficient incentives, the social planner can fix the problem by criminalizing the negligent behavior and setting the punishment to equal the harm multiplied by the multiplicative inverse of the level of under-enforcement. If the burden of proof in criminal trials is set at 95% (beyond a reasonable doubt) then, at least in some circumstances, the cumulative effect of criminal and private enforcement will create optimal incentives.