The impact of deductibles and partial coverage on the care level chosen by risk-neutral injurers with (mandatory) liability insurance.

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1. Introduction

This paper analyzes the (individually) optimal choice of unilateral care by a risk-averse injurer who is insured. Even risk-averse agents might buy insurance for other reasons than risk-allocation (see risk-management text book, Skogh on pooling, or Kirstein 2000 or Kirstein/Kirstein/Gerhardt (2009) on strategic insurance).

E. Landes (1982, 51) claims that the analysis of insurance in this context has to be carried out under the assumption of risk-aversion instead of risk-neutrality. However, Kirstein 2000 has argued that strategic effects of insurance can be studied separately from its mere risk-allocation effects when maintaining the risk-neutrality assumption. This paper follows the latter approach.

Moreover: for driving a car (and other activities), liability insurance is mandatory in many countries even if the owner is risk-neutral (i.e. a firm). Main rationale is perhaps to avoid limited liability.

The Law and Economics literature often assumes that insurance reduces the injurer’s incentive to invest into precaution:

Cooter/Ulen (textbook p. 355): Insurance companies employ various means to reduce moral hazard, notably co-insurance, deductibles, and experience rating….While these devices reduce moral hazard, they cannot eliminate it. Consequently, insurance inevitably undermines the insured’s incentives for precaution.

More cautiously Ambrose/Carroll/Regan (2013), p. 316: Some scholars argue that the existence of liability insurance undermines...the deterrence effect of tort liability....(because) cost arising from a liability claim are at least partially covered by the liability insurer......(Thus, insured injurers) may be less likely to take care to prevent others from harm.

Ambrose/Carroll/Regan (2013), p. 321: ...optimal co-payments would include a deductible, a coinsurance rate, or both in the single period case. In the multi-period case, the optimal policy is priced based on the level of care taken (with a reference to Danzon 1985 on medical malpractice).

Experience rating consists of an increase in the insurance fee if a claim has been made in a previous period. As the increase in the fee can be expressed as an annuity or perpetuity, its present value
should have a similar effect as a deductible paid only once and right after the accident. Hence, this paper limits its focus on the combination of co-insurance (or: partial coverage) and deductible.

This paper studies the impact of deductibles and co-insurance on the care choice.

To keep matters simple, this paper looks at single-sided care cases only.

Risk-Management textbook: car insurance is often characterized by limited coverage (which is excluded from consideration in this paper).

2. Related literature

3. The model

3.1 Setup

Consider a risky activity carried out by a single decision maker, the (prospective) injurer, denoted M. Carrying out the activity may lead to an accident, accompanied by harm to others. M can reduce he harm H by exercising care x (we do not distinguish here between increasing care or reducing the level of the dangerous activity).

Let H(x) be continuous and twice differentiable, and decreasing in x, but at a decreasing rate, hence: H’<0<H”. Exercising care is costly for M; the cost are written as c(x) with c’>0≥c”.

M is held strictly liable, but can buy insurance from an insurance company N. If M is insured and an accident occurs, then he can claim coverage. In that case, he bears only a share of the harm, denoted as a, with 0<a<1, and has to pay a deductible D≥0 in addition to aH(x). If he does not turn to the insurer for coverage, he bears the full harm H(c); in other words: enforcement is assumed to be costless and error-free (see the literature cited in Ambrose/Carroll/Regan(2013, 321f.) for the analysis of positive enforcement costs and court errors).

An accident is assumed to occur with probability q, which is independent of care.

Many contributions to the economic analysis of tort instead made the assumption that q depends on care, whereas H is often assumed to be independent of care. The latter assumption would, however, make a deductible impractical. After an accident, the injurer would choose coverage if the deductible is smaller than then actual (not the expected) harm H, and bear H otherwise. If H is exogenously given, then this decision is trivial.

If, however, H can be influenced by the choice of care, a meaningful decision problem arises. For the sake of simplicity this paper models only H as a function of care, not both H and q.
Figure 1 shows M’s decision problem after having made a contract with the insurer. He first has to choose a non-negative care level $x$; then a random move decides whether an accident occurs (with $q$) or not ($1-q$). If no accident occurs, M only has to bear care cost $c(x)$. In case of an accident M has to decide whether to claim coverage – in that case his payoff is $-aH(x)-D-c(x)$, or not, in which case he bears $H(x)-c(x)$.

In the following model, thus, the exogenous parameters are $q$, $a$, and $D$, whereas $x$ (and the decision to claim coverage) are endogenous.

*It would perhaps be insightful to add a bargaining stage between M and N, so as to check whether/under which conditions it is sensible for the insurer to enter the contract with M.*

### 3.2 Efficient care

Regardless of how N and M distribute the damage among them (by means of $a$ and $D$), the socially optimal care level in this single-sided care case minimizes sum of the expected harm and the care cost.

Denote the social optimum as $x^*$, then (Brown 1973, Posner, Shavell 1980):

1. $x^* = \text{argmin} \{qH(x)+c(x)\}$.

The first-order condition for an internal minimum is

2. $qH'+c'=0 \iff -qH'=c'$.

In case of an internal minimum the second-order condition is satisfied as $H''+c''>0$. 
3.3 M’s individually optimal decisions

M’s choice of care (at the beginning of the decision tree) may depend on whether he expects himself to claim coverage or not (at the second decision node). Hence, we first analyze the later decision.

3.3.1 To claim or not to claim coverage

At the final decision node, given his previous choice of a care level, M chooses to claim coverage if

\[(3) \ aH(x)+D+c(x) \leq H(x)+c(x)\]

which is equivalent to

\[(4) \ D \leq (1-a)H(x) \Leftrightarrow H(x) \geq D/(1-a).\]

M abstains from claiming coverage if \(D > (1-a)H(x)\). In that case, he pays the insurer’s part of the harm so as to avoid having to pay the deductible.

Figure 2: Efficient care

Since \(H(x)\) is decreasing in \(x\), it is obvious that M, if he has chosen a low care level in the beginning, is more inclined to demand coverage, whereas a high care level will induce him to bear the damage. A level of care exists at which M is indifferent between insurance coverage and bearing the full harm. This threshold care level is a function of the parameters \(D\) and \(a\), and can be determined as

\[(5) \ x(D,a) = H^{-1}(D/(1-a)).\]
Figure 3 visualizes M’s choice problem at the terminal decision node: The threshold $x(D,a)$ is found where the horizontal line representing $D$ intersects with the curve $(1-a)H(x)$.

**Figure 3: Decision between claiming coverage ($x<x(D,a)$) and bearing harm ($x>x(D,a)$)**

For $D>0$ and $a<1$ the hyperbolic shape of $H(x)$ with $H(x)>0$ for all $x>0$ makes sure that $x(D,a)>0$.

### 3.3.2 Individually optimal care if M later wants to bear the accident cost

The choice of the optimal care level is not only influenced by the probability with which an accident occurs, namely $q$, but also by the decision whether or not to claim coverage from the insurer, which is guided by (4). Thus, we first analyze the situation if M initially has chosen a care level high enough to later induce him to bear the harm in case of an accident, i.e., the care levels under scrutiny exceed $x(D,a)$. Afterwards, we analyze the optimal decision if the initially chosen care falls short of $x(D,a)$.

If M has initially chosen a care level high enough so that he expects himself to bear the harm should an accident actually occur, then his optimal choice seeks to minimize the expected harm plus the care cost

$$(5) \quad qH(x)+c(x) \text{ s.t. } x \geq x(D,a).$$

The FOC for an internal solution in this case is

$$(6) \quad qH'+c'=0,$$

which is identical to the social problem. Hence, if $x^*>x(D,a)$ then the local minimum is characterized by $-qH'=c'$, which is identical to (2) and, therefore, the local minimum is equal to $x^*$. 
If, however, \( x^* \) is smaller than \( x(D,a) \), then the left hand side of (6) is negative for \( x \geq x(D,a) \). In that case, \( M \) would want to reduce \( x \) as far as possible. Under the constraint \( x \geq x(D,a) \), the solution of (5) would be to choose \( x(D,a) \). In fact, \( M \) would prefer to choose an even smaller care level, but that would alter his later decision whether to claim insurance, and another yield function would become relevant.

This leads to a first intermediate result:

**Lemma 1:** Assume that \( M \) considers choosing a care level that exceeds \( x(D,a) \) and, therefore, expects to bear the harm should an accident occur. Then

a) If \( x^* \geq x(D,a) \), then the optimal care choice would be \( x=x^* \).

b) If \( x^* < x(D,a) \), then \( M \) would prefer to choose \( x=x(D,a) \) over any \( x > x(D,a) \).

c) \( x^* \) is positive for \( q>-c'/H(0)' \).

**Proof:** All three parts are straightforward.

### 3.3.3 Individually optimal care if \( M \) later claims insurance coverage

\( M \) may as well consider choosing initially a care level smaller than \( x(D,a) \). In that case \( M \) would ask the insurer to cover the harm. \( M \) then had to bear \( aH(x)+D \), and his care choice (denoted as \( x^\# \)) is, thus, given by

\[
(7) \quad x^\# = \text{argmin} \{q[aH(x)+D]+c(x)\} \text{ s.t. } x<x(D,a).
\]

The FOC now is

\[
(8) \quad aqH'+c'= 0 \Leftrightarrow -aqH'=c'.
\]

Since \( a<1 \), the comparison of (8) with (6) and (2) implies that care would deviate from the social optimum. This leads to

**Lemma 2:** Assume \( M \) considers choosing a care level smaller than \( x(D,a) \), which induces him to claim coverage if an accident occurs. Then one of three cases may occur:

a) If \( x(D,a) > 0 \) and the left hand side of (8) is positive for \( x=0 \), then \( M \) would prefer to choose \( x=0 \).

b) If the left hand side of (8) is negative at \( x(D,a) \), then an internal minimum does not exist in the interval under scrutiny, and \( M \) would prefer to choose a care level greater than \( x(D,a) \).

c) If a local minimum exists for \( 0 < x < x(D,a) \), then \( M \)'s optimal care choice \( x^\# \) is characterized by 
\[-H'=c'/aq.\]

d) If \( a<1 \) then \( x^\# < x^* \).

**Proof:** In case a) the local minimum \( x^\# \) is negative and, hence, not feasible. Thus, the corner solution \( x=0 \) is the minimum. In b) the local minimum \( x^\# \) exceeds \( x(D,a) \), but choosing an \( x \) greater than \( x(D,a) \) would induce \( M \) to bear the harm. The optimal care level \( x \) that still induces him to ask for coverage is the corner solution \( x(D,a) \). Case c) is straightforward. To prove d), recall that \( H'<0<c' \) and \( q>0 \), and compare equations (6) \(-qH'=c' \) characterizing \( x^* \) and (8) \(-aqH'=c' \) characterizing \( x^\# \).
3.3.4 Positive result: optimal decisions of M

Define \( h(y) = H^{-1}(y) \). Recall that \( x(D,a) = h(D/(1-a)) > 0 \) and \( c(0) = 0 \). Further recall that \( x^* = h'(-c'/q) \) and \( x^# = h'(-c'/aq) \). Then \( 0 < q, a < 1 \) implies that \( x^# < x^* \) as \( H \) and \( h \) are downwards sloped. This leads to the core result of this paper.

**Proposition 1:** Given the decision problem of an insured injurer as outlined in figure 1, with parameters \( a \) with \( 0 \leq a < 1 \), \( D > 0 \), and \( q \) with \( 0 < q < 1 \), and care technology \( H(x) \), \( c(x) \) with \( x \geq 0 \). Denote \( a^# = [qH(x^*) - qD + c(x^*) - c(x^#)]/qH(x^#) \) and \( a_0 = [qH(x^*) - qD + c(x^*)]/qH(0) \).

Assume first \( x^# > 0 \), then

1. if \( x(D,a) < x^# \) then M chooses \( x = x^* \) and bears the harm if an accident occurs;
2. if \( x^# < x(D,a) < x^* \) then M chooses
   2.1. \( x = x^* \) and bears the harm if \( a > a^# \) or
   2.2. \( x = x^# \) and claims coverage if \( a < a^# \);
3. if \( x^* < x(D,a) \) then M chooses \( x = x^# \) and claims coverage.

Now assume \( x^* > 0 \geq x^# \), then

4. if \( x(D,a) < 0 \) then M chooses \( x = x^* \) and bears the harm;
5. if \( 0 < x(D,a) < x^* \) then M chooses
   5.1. \( x = x^* \) and bears the harm if \( a > a_0 \) or
   5.2. \( x = 0 \) and claims coverage if \( a < a_0 \);
6. if \( x^* < x(D,a) \) then M chooses \( x = 0 \) and claims coverage.
7. In case of \( x^* < 0 \) M will choose \( x = 0 \) and
   7.1. bears the harm if \( D > (1-a)qH(0) \), or
   7.2. claims coverage if \( D < (1-a)qH(0) \).

**Proof:** In case 1 any feasible \( x \) exceeds \( x(D,a) \). Hence, M will bear the harm, and chooses \( x^* > 0 \). In case 2, if M chose some \( x > x(D,a) \) then he would bear the harm, so \( x^* \) would be optimal. If, however, he chose \( x < x(D,a) \) then he would ask for coverage and \( x^# \) would be optimal. The payoff from the former option exceeds the latter if \( qH(x^*) + c(x^*) < q[aH(x^#) + D] + c(x^#) \), which is equivalent to \( a > a^# \). In case 3, both feasible local optima fall short of \( x(D,a) \) so only \( x^# \) (and claiming coverage) is relevant. The proof for cases 4-6 follows the same line of argumentation, but now with \( x = 0 \), \( H(0) \), and \( c(0) = 0 \). In case 7, no local optimum is feasible, so \( x = 0 \) is optimal. To bear the damage is better if \( qH(0) < aqH(0) + D \), which is equivalent to \( D < (1-a)qH(0) \): Q.E.D.

3.4 Normative result: efficient insurance contracts

**Corollary:** a) M chooses \( x = x^* \) and bears the harm if

- \( x(D,a) < x^# < x^* \)
- \( 0 < x^# < x(D,a) < x^* \) and \( a > a^# \)
- \( x^#, x(D,a) < 0 \)
- \( x^# < 0 < x(D,a) < x^* \) and \( a > a_0 \)

There are \( (D,a) \) combinations with \( D < H(x^*) \) and \( a < 1 \) which induce M to choose efficient care even though he is partially insured.
Can the information in the corollary somehow be reorganized to formally prove the conditions under which the following proposition holds?

**Proposition 2:** Contract combinations \((D,a)\) exist such that the insured injurer is induced to choose efficient care \(x^*\).

### 4. Simulation

#### 4.1 Setup

The simulation is based on a specific harm technology \(H(x)\), namely \(H(x)=1/(1+x)\), thus \(H'=x/(1+x)^2\). Denote the inverse of \(H(x)\) as \(h(H)\), with \(h(H)=(1-H)/H\), thus \(h'=\frac{-H-(1-H)}{H}=-1/H^2\).

Recall that \(x(D,a) = h(D/(1-a)) > 0\), hence \(x(D,a)=(1-a-D)/D\) can be positive (if \(1-a>D\)) or negative.

The injurer’s cost of harm are set to \(c(x)=x/10\), thus \(c'=1/10\) and \(c(0)=0\). Recall that \(x^* = h'(-c'/q) = (10q)^{-0.5}\) and \(x^\# = h'(-c'/aq) = (10aq)^{-0.5}\).

We first present simulation results for \(q \in \{0.2; 0.4; 0.6; 0.8\}\) and only look at modifications of \(D\) and \(a\). The case of \(q<0.1\) (implying \(x^*<0\) so that \(x^*\) is not feasible) will be discussed separately.

The simulation pursues two goals:

- Check results of theoretical derivation (leading to figures for the relevant cases with kinked liability curve (simulation))
- Identify incentive compatible \((D,a)\) combinations.

We will discuss the case \(q=0.4\) in greater detail, and limit with regard to the other cases the focus on the derivation of incentive compatible \((D,a)\) combinations.

#### 4.2 Simulation results for \(q=0.4\)

\(q=0.4\) implies that \(x^*=1\) and \(x^\# = h'(-c'/a) = (10a)^{-0.5}\).

Regarding the first goal of the simulation, table 1 presents the main results for \(q=0.4\) and for several \((D,a)\)-combinations, namely \(a=0.2, 0.4, 0.4\) and \(D=0.2, 0.4, 0.6, 0.9\). These parameter settings will cover almost all cases mentioned in proposition 1 (only the case of negative \(x^*\) is missing).

The respective \(a\)-value determines the respective \(x^\#\), which is indicated in the second row of the table. Knowing \(x^*\) and \(x^\#\) allows for the derivation of the respective values of \(a^\#\) and \(a_0\), which are relevant if the parameter combination addresses cases 2 or case 5.

Each \((D,a)\) combination leads to a value for \(x(D,a)\), indicated in each cell in the top row, and to an optimal choice \(x \in \{0; x^*; x^\#\}\), indicated in the second row of each cell. Finally, the cells also report which case of Proposition 1 is addressed here.

In bold are the cases in which \(M\)’s optimal decision depends on the comparison of \(a\) (upper row) with \(a_0\) (cases 5.1 vs. 5.2) or \(a^\#\) (cases 2.1 vs. 2.2).

Table 1 indicates that \((D,a)\) combinations exist which are incentive compatible, i.e., they induce \(M\) to choose the socially optimal care level \(x^*\) (examples are highlighted in yellow in the table). The reasons for the optimal choice may differ (cases 1, 2.1, 4, and 5.1 are listed as incentive compatible).
The evaluation of a series of simulations for \( q=0.4 \) supports the result of proposition 2.

For \( q=0.4 \), table 1 outlines the minimal required value of \( a \), given the value of \( D \), that induces the insured injurer to choose optimal care.

Repeated simulations (for \( q=0.4 \)) demonstrate that a negative relation exists between \( D \) and the minimum \( a \) which is still incentive compatible. In figure 4, \( D \) is on the horizontal axis, whereas the minimal \( a \) is on the horizontal axis.

**Figure 4:** \((D,a)\)-combinations that induce \( M \) to choose \( x^* \) if \( q=0.4 \)

The dots indicated in figure 4 are connected by a line which approximately is described by the equation

<table>
<thead>
<tr>
<th>( D )</th>
<th>( D ) = 0.2</th>
<th>( a ) = 0.3</th>
<th>( a ) = 0.4</th>
<th>( a ) = 0.5</th>
</tr>
</thead>
<tbody>
<tr>
<td>( a_0 = 0.3-D = 0.1 )</td>
<td>( x(D,a) = 3 ) x = 0</td>
<td>( x(D,a) = 2.5 ) x = ( x^* ) case 3</td>
<td>( x(D,a) = 2 ) x = ( x^* ) case 3</td>
<td>( x(D,a) = 1.5 ) x = ( x^* ) case 3</td>
</tr>
<tr>
<td>( a_0 = -0.1 )</td>
<td>( x(D,a) = 1.25 ) x = 0</td>
<td>( x(D,a) = 0.75 ) x = ( x^* ) case 2.2</td>
<td>( x(D,a) = 0.5 ) x = ( x^* ) case 2.2</td>
<td>( x(D,a) = 0.25 ) x = ( x^* ) case 2.1</td>
</tr>
<tr>
<td>( a_0 = 0.25 )</td>
<td>( x(D,a) = 0.6 ) x = 0</td>
<td>( x(D,a) = 0.4 ) x = ( x^* ) case 5.2</td>
<td>( x(D,a) = 0.2 ) x = ( x^* ) case 2.1</td>
<td>( x(D,a) = 0.1 ) x = ( x^* ) case 1</td>
</tr>
<tr>
<td>( a_0 = -0.3 )</td>
<td>( x(D,a) = 0.333 ) x = ( x^* ) case 5.1, as ( a &gt; a_0 )</td>
<td>( x(D,a) = 0.1667 ) x = ( x^* ) case 2.1</td>
<td>( x(D,a) = 0 ) x = ( x^* ) case 1</td>
<td>( x(D,a) = -0.1667 ) x = ( x^* ) case 1</td>
</tr>
<tr>
<td>( a_0 = -0.4 )</td>
<td>( x(D,a) = -0.11 ) x = ( x^* ) case 4</td>
<td>( x(D,a) = -0.22 ) x = ( x^* ) case 1</td>
<td>( x(D,a) = 0 ) x = ( x^* ) case 1</td>
<td>( x(D,a) = -0.44 ) x = ( x^* ) case 1</td>
</tr>
</tbody>
</table>
(9) \( a = 1 - 4D/3 \).

Hence, \((D, a)\) combinations above this line are incentive compatible (for \(q = 0.4\)), whereas contracts below this line would induce \(M\) not to choose \(x^*\) (but either \(x^n\) or 0 instead).

**Table 2: Minimal incentive compatible \(a\)-values if \(q = 0.4\)**

<table>
<thead>
<tr>
<th>(D)</th>
<th>(M) chooses (x^*) if (a) is greater than</th>
</tr>
</thead>
<tbody>
<tr>
<td>0,01</td>
<td>0,981</td>
</tr>
<tr>
<td>0,1</td>
<td>0,81</td>
</tr>
<tr>
<td>0,2</td>
<td>0,64</td>
</tr>
<tr>
<td>0,3</td>
<td>0,49</td>
</tr>
<tr>
<td>0,4</td>
<td>0,36</td>
</tr>
<tr>
<td>0,5</td>
<td>0,25</td>
</tr>
<tr>
<td>0,6</td>
<td>0,16</td>
</tr>
<tr>
<td>0,7</td>
<td>0,05</td>
</tr>
<tr>
<td>0,751</td>
<td>0</td>
</tr>
</tbody>
</table>

### 4.3 \(q = 0.2\)

\(q = 0.2\) implies \(x^* = h'(-c'/0.2) = h'(-0.5) = 2^{-0.5-1} = 0.4142...\) and \(x^n = h'(-c'/aq) = (2a)^{-0.5-1}.\)
0.05  0.9305
0.1   0.8635
0.2   0.7371
0.3   0.62
0.4   0.5143
0.5   0.414
0.6   0.3142
0.7   0.2142
0.8   0.1145
0.9   0.0142
0.95  0

Approx: \( a = 1 - 1.4D \)

### 4.4 q=0.6

Simulation result for \( q=0.6 \)

![Graph showing \( q=0.6 \) parameter settings]

**Optimal parameter settings if \( q=0.6 \)**

<table>
<thead>
<tr>
<th>( D )</th>
<th>( x^* )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.05</td>
<td>0.8812</td>
</tr>
<tr>
<td>0.1</td>
<td>0.77</td>
</tr>
<tr>
<td>0.2</td>
<td>0.5701</td>
</tr>
<tr>
<td>0.3</td>
<td>0.4001</td>
</tr>
<tr>
<td>0.4</td>
<td>0.2602</td>
</tr>
<tr>
<td>0.5</td>
<td>0.1498</td>
</tr>
<tr>
<td>0.6</td>
<td>0.0498</td>
</tr>
<tr>
<td>0.7</td>
<td>0</td>
</tr>
</tbody>
</table>

Approx: \( a = 1 - 1.42D \) for \( D < 0.7 \)
4.5 q=0.8

Simulation results for q=0.8

Approx $a = 1 - 2D$ for $D < 0.6$

4.6 Comparison between simulation results

$q = 0.2 \Rightarrow a = 1 - 1.4D$

$q = 0.4 \Rightarrow a = 1 - 1.33D$. 

$q = 0.6 \Rightarrow a = 1 - 1.42D$

$q = 0.8 \Rightarrow a = 1 - 2D$

The greater $q$, the greater the tradeoff between $D$ and $a$ (the quicker is the decline in the need of one contract parameter if the other parameter increases).
4.7 The case $q<0.1$

If the accident probability $q$ falls short of 0.1, then even $x^*$ is negative. This addresses case 7 of the proposition 1. The case 7.1 would require $D$ to be greater than 0.24, whereas case 7.2 prevails if $D<0.24$.

5. Discussion and policy implications

What are the insurer’s payoffs in the various cases? In case of mandatory insurance, however, we don’t really have to discuss the bargaining problem between injurer and insurer (at most the participation constraint of the insurer).

Literature

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