Minimum Quality Standards and Exports

Laura Birg* Jan S. Voßwinkel**

April 2018

Abstract

This paper studies the interaction of a minimum quality standard and exports in a vertical product differentiation model when firms sell global products. If ex ante quality of foreign firms is lower (higher) than the quality of exporting firms, a mild minimum quality standard in the home market hinders (supports) exports. The minimum quality standard increases quality in both markets. A welfare maximizing minimum quality standard is always lower under trade than under autarky. A Minimum quality standard reduces profits for the exporting firm. It increases domestic welfare, but reduces welfare in the export market.

JEL Classification: F12, L13, L50

Keywords: minimum quality standard, vertical differentiation, exports

1 Introduction

This paper studies the interaction of a minimum quality standard and exports in a vertical product differentiation model where firms sell global products. In particular, it analyzes the effect of a (national) minimum quality standard on export volume and welfare as well as the effect of exports on standard setting.

Governments regularly impose minimum quality standards on products to avoid externalities or because of paternalistic reasons. An additional reason for minimum quality standards may be that oligopolistic firms tend not to choose welfare maximizing quality levels of their products (Scarpa, 1998). In these settings, minimum quality standards are welfare increasing.

*Department of Economics, University of Göttingen, Platz der Göttinger Sieben 3, 37073 Göttingen, Germany, laura.birg@wiwi.uni-goettingen.de.
**Department of Economics, NGU Nürtingen-Geislingen University, Neckarsteige 6-10, 72622 Nürtingen, Germany, jan.vosswinkel@hfwu.de.
An example for minimum quality standards is the product design policy of the European Union. Based on the Ecodesign Directive 2009/125/EC, for instance, the European Commission sets minimum energy efficiency standards for many household appliances such as dish washers, vacuum cleaners, light bulbs and television sets etc. In addition, the EU sets minimum quality standards with regard to CO$_2$ emissions for new passenger cars [Regulation (EC) No 443/2009]. These energy efficient standards and emissions standards apply for all products sold on the European market.

But as (European) firms also export their products to markets, where those standards do not apply, the minimum quality standard may result in spillovers to those markets. This is the case if firms sell global goods, goods with global (instead of country specific) quality levels, such as passenger cars, computers or television sets. If a domestic quality standard affects prices and (potentially) quality levels in another country, it also might affect foreign producers by altering competition and affecting quality investment decisions. It might also affect consumers and may result in an increased or decreased consumer surplus. This is why the regulator should not only consider the effects of a minimum quality standard on domestic consumers and producers. If domestic minimum quality standards affect foreign producers and consumers, this effect should also be taken into account.

The international activities of firms may also affect the standard-setting behavior of a regulator. If higher domestic standards promote exports, the regulator may have an incentive to increase quality standards; vice versa, if higher standards hinder exports, there may be an incentive to decrease standards. We show that international trade has an influence on welfare-maximizing minimum quality standards.

One strand of literature on trade and minimum quality standards focuses on standards that are applied by importing countries. In this context minimum quality standards are seen as potential trade barriers. Minimum quality standards may lead to higher cost for foreign firms compared to domestic firms by design, by administration or compliance cost (Henson & Jaffee, 2008, Marette & Beghin, 2012; Baltzer, 2011).

Another strand of literature analyzes the effects of domestic minimum quality standards on exports, commonly finding positive effects for the exporting firm. Minimum quality standards may facilitate the signaling of product quality levels. (Leland, 1979, Hudson & Jones, 2003). Mangelsdorf et al. (2012) show that voluntary and mandatory standards have a positive effect on Chinese food exports. Their estimation results indicate that this positive effect increases if the national standards are harmonized to international standards. Clerides & Hadjiyiannis (2005) provide some evidence that minimum quality standards on used goods may affect trade flows from high to low standard
countries.

The literature on standards and trade has also analyzed the effect of trade on standard setting. Petropoulou (2013) analyzes the effect of trade on unilateral and multilateral standard-setting. Trade induced externalities leads to inefficient high or low national standards. Harmonized multilateral standards may be beneficial, but cooperation between countries might be restricted, if countries differ and lump-sum transfers are restricted. Lutz & Pezzino (2012) find that mutual recognition of standards always increases welfare.

This paper studies the interaction of a minimum quality standard and exports in a vertical product differentiation model where firms sell global products. Global products are goods that are associated with a global quality level such as passenger cars, smartphones, or computers. A reason for selling global products could be very high additional costs of quality differentiation or a global brand image. In our model, this implies that the exported good is of the same quality as the good sold in the domestic market.

We use a setting of three firms, so that there is duopoly in both countries under trade.\(^1\)

Like Ronnen (1991) and Crampes & Hollander (1995), we consider duopolistic markets, where single product firms face minimum quality standards as exogenous constraints. First we analyze the effect of an exogenous minimum quality standard on prices, export volume, and welfare. We later then endogenize the quality standard. We assume that the provision of quality improvements entails variable costs, similar to Crampes & Hollander (1995) and Petropoulou (2013). Quality improvements are reached by using higher quality materials or other variable factors in the production process. Like Baltzer (2011), this paper does not assume that the minimum quality standard is implicitly or explicitly discriminating in nature. Firms are assumed to have identical costs for quality improvements.

We show that depending on the quality level in the export market, a national minimum quality standard may hinder or promote exports. International trade always leads to a lower welfare maximizing minimum quality standard compared to autarky. The minimum quality standard increases welfare in the home country, but lowers welfare in the export country. It always lowers profits for the exporting firm.

The rest of the paper is organized as follows. In the next section, the vertical differ-

\(^1\)With only one firm in each country, a minimum quality standard in the home market would translate directly to the quality of the exporting (monopoly) firm and the exported good. In such a setting, the home government could control directly the quality of domestic products in foreign markets. Since we are more interested in the (unintended) externalities of domestic minimum quality standards, we choose a setting with two firms in the home country.
entiation model is presented and the effect of a standard is analyzed. Section 3 studies the case of a comparatively low quality level in the export market, section 4 studies a comparatively high quality level in the export market. Section 5 concludes.

2 The Model

Consider two countries, \( j = H, F \) (home, foreign), in which firms sell global products. In country \( H \), the market structure is duopolistic: Two firms, \( A \) and \( B \) supply a good in two quality levels \( \omega_A \) and \( \omega_B \), with \( \omega_A > \omega_B \). In country \( F \), a local firm \( C \) produces a good with quality level \( \omega_C \).

Additionally, firms from country \( H \) may export to country \( F \). In line with the trade literature (e.g. Crozet, Head & Mayer, 2009), we assume that (only) the high quality firm \( A \) exports to \( F \).

Assume that firms sell global products. This is, firms choose global quality levels and do not differentiate products across markets. This implies that the exported good is of the same quality as the good sold in the domestic market (\( \omega_{A,H} = \omega_{A,F} \)). This might be the case if quality differentiation would impose very high additional costs, e.g. investment or setup costs. This might also be the case if a brand image commits a firm to a worldwide uniform quality level.

We use a setting of three firms, so that there is duopoly in both countries under trade.\(^2\)

In both countries, the production technology is characterized by variable cost, which are convex in quality and linear in quantity \( c_i = \omega_i^2 q_i, i = A, B, C \). Quality improvements are reached by using higher quality materials or other variable factors such as high skilled workers or by using more of some variable factors, such as labor or energy in the production process. Alternatively, assume that the production process is more costly because of a higher degree of complexity or higher hygienic standards.

In both countries, consumers are heterogeneous with respect to their gross valuation of quality, represented by a parameter \( \theta \). In both countries, \( \theta \) is uniformly distributed on the interval \([a, b]\) with \( b = a + 1 \).\(^3\) The consumer heterogeneity can be interpreted

\(^2\)With only one firm in each country, a minimum quality standard in the home market would translate directly to the quality of the exporting (monopoly) firm and the exported good. In such a setting, the home government could control directly the quality of domestic products in foreign markets. Since we are more interested in the (unintended) externalities of domestic minimum quality standards, we choose a setting with two firms in the home country.

\(^3\)Assume \( b \geq b_{\text{min}} = \frac{3}{2} \) to guarantee equilibrium existence.
as differences in income\(^4\) or as differences in consumption patterns. Frequent usage may be accompanied by a higher willingness to pay for quality. We assume that demand is identical in both countries. The asymmetric effect of the minimum quality standard in both countries is not based on asymmetric demand.

Each consumer buys at most one unit of the most preferred good. The utility derived from no purchase is zero, while a consumer who buys one unit of the good obtains a net utility of

\[ U = \theta \omega_i - p_i, \ i = A, B, C. \]  

In country \(H\), the marginal consumer indifferent between purchasing the high quality good and the low quality good is

\[ \theta^* = \frac{p_A - p_B}{\omega_A - \omega_B}. \]  

In \(H\) demand is given as

\[ q_{A,H} = b - \frac{p_{A,H} - p_{B,H}}{\omega_A - \omega_B}, \ q_{B,H} = \frac{p_{A,H} - p_{B,H}}{\omega_A - \omega_B} - a. \]  

Consider two cases. In the first case, the quality of the local firm in the foreign market is low, i.e. \(\omega_A > \omega_C\). In the second case, ex ante quality of the local firm in the foreign market is high, i.e. \(\omega_A < \omega_C\). Both cases may be relevant for the EU, because European firms export their products to high-quality markets such as Japan or the USA as well as to emerging markets with a lower average quality level for some products.

If \(\omega_A > \omega_C\), demand in country \(F\) is

\[ q_{A,F} = b - \frac{p_{A,F} - p_{C,F}}{\omega_A - \omega_C}, \ q_{C,F} = \frac{p_{A,F} - p_{C,F}}{\omega_A - \omega_C} - a. \]  

If \(\omega_A < \omega_C\), demand in country \(F\) is

\[ q_{A,F} = \frac{p_{C,F} - p_{A,F}}{\omega_C - \omega_A} - a, \ q_{C,F} = b - \frac{p_{C,F} - p_{A,F}}{\omega_C - \omega_A} - a. \]  

Firms’ profits are

\[ \pi_i = (p_i - \omega_i^2) q_i. \]  

Trade cost are normalized to zero.

We first study the effect of an exogenous minimum quality standard on quality levels,

\(^4\)Note that \(\theta\) can also be interpreted as the marginal rate of substitution between income and quality (see Tirole, 1988).
prices, and welfare. Then we endogenize the minimum quality standard, assuming a simultaneous game between a welfare maximizing government and the firms.

Competition follows a two-stage game: In the first stage, firms choose quality levels. In the case of minimum quality standard, the government sets a minimum quality standard. In the second stage, firms compete in prices.

2.1 Low Quality Level in the Export Market

Assume $\omega_A > \omega_C$, i.e. the export market is characterized by a relatively low quality level.

2.1.1 Equilibrium without Government Intervention

Quality levels, prices, and quantities under no regulation can be found in the Appendix.

2.1.2 Exogenous Minimum Quality Standard

Consider now the introduction of a mild exogenous minimum quality standard $\Omega > \omega_B$ in $H$. Quality levels, prices, and quantities under the minimum quality standard can be found in the Appendix. Assume that $\Omega$ is binding for the low quality firm $B$. Firm $A$ increases its quality level in response to the mild minimum quality standard ($\omega_A^\Omega > \omega_A$).

In $F$, firm $C$ now faces competition from the increased quality of products of firm $A$ and increases the quality of its product ($\omega_C^\Omega > \omega_C$). So there is an external effect of the domestic minimum quality standard on the quality of available products in $F$.

In $H$, prices increase ($p_{A,H}^\Omega > p_{A,H}$, $p_{B,H}^\Omega > p_{B,H}$). In $F$, prices also increase ($p_{A,F}^\Omega > p_{A,F}$, $p_{C,F}^\Omega > p_{C,F}$).

In $H$, the minimum quality standard shifts demand from the high quality firm $A$ to the low quality firm $B$ ($q_{A,H}^\Omega < q_{A,H}$, $q_{B,H}^\Omega > q_{B,H}$). In $F$, the minimum quality standard also shifts demand from the high quality firm $A$ to the low quality firm $C$ ($q_{A,F}^\Omega < q_{A,F}$, $q_{C,F}^\Omega > q_{C,F}$). The minimum quality standard decreases the volume of exports.

**Proposition 1** Suppose that $\omega_A > \omega_C$ and that a mild minimum quality standard is introduced. In $H$, the minimum quality standard shifts demand from the high-quality firm $A$ to the low quality firm $B$. In $F$, the minimum quality standard shifts demand from the high-quality firm $A$ to the low quality firm $C$. This is, the minimum quality standard reduces the volume of exports.
2.1.3 Endogenous Minimum Quality Standard

Consider now the introduction of an endogenous minimum quality standard in $H$, with the government setting the minimum quality standard to maximize domestic welfare. Variables under autarky are denoted by an asterisk.

**Autarky** Consider first the case of autarky, where firm $A$ only sells in $H$. The government sets the minimum quality standard $\Omega^*$ to maximize national welfare, the sum of consumer surplus and profits: $W_H^* = CS_H^* + \pi_A^* + \pi_B^*$. This is equivalent to the case described in Ecchia & Lambertini (1997). Quality levels, prices, and quantities can be found in the Appendix.

**Exports** Consider now the case of trade, where firm $A$ exports to country $F$. If the government maximizes national welfare $W_H (W_H = CS_H + \pi_A + \pi_B)$ the minimum quality standard under trade is lower than under autarky ($\Omega < \Omega^*$). The quality level of firm $A$ is also lower under trade than under autarky ($\omega_A^O < \omega_A^O*$).

If the government in $H$ maximizes global welfare ($W = CS_H + CS_F + \pi_A + \pi_B + \pi_C$), the minimum quality standard is also lower than under autarky ($\Omega < \Omega^*$).

**Proposition 2** Suppose that $\omega_A > \omega_C$ and that an endogenous minimum quality standard is introduced in country $H$. Then the welfare maximizing standard is lower under trade than under autarky.

2.1.4 Welfare

Consumer surplus, profits, and welfare can be found in the Appendix. The minimum quality standard increases consumer surplus in $H$ ($CS_H^{\Omega} > CS_H$), decreases profits of firm $A$ ($\pi_A^{\Omega} < \pi_A$) and increases profits of firm $B$ ($\pi_B^{\Omega} > \pi_B$). Consumer surplus, profits, and welfare can be found in the Appendix. The minimum quality level increases welfare if it is sufficiently low ($W_H^{\Omega} > W_H$). In $F$, the minimum quality standard decreases consumer surplus ($CS_F^{\Omega} < CS_F$), but increases profits of firm $C$ ($\pi_C^{\Omega} > \pi_C$). The minimum quality standard reduces welfare in $F$ ($W_F^{\Omega} < W_F$).

**Proposition 3** Suppose that $\omega_A > \omega_C$ and that a mild minimum quality standard is introduced. In $H$, the minimum quality standard decreases profits of the high-quality firm $A$ increases profits of the low-quality firm $B$. The minimum quality standard increases welfare. The minimum quality standard reduces welfare in $F$.
2.2 High Quality Level in the Export Market

Assume $\omega_A < \omega_C$, i.e. the export market is characterized by a relatively high quality level. Assume that only $A$ exports its products, but $C$ does not export its products to $H$.

2.2.1 Equilibrium without Government Intervention

Quality levels, prices, and quantities can be found in the Appendix.

2.2.2 Exogenous Minimum Quality Standard

Consider the introduction of an exogenous mild minimum quality standard $\Omega > \omega_B$ in $H$. Quality levels, prices, and quantities can be found in the Appendix. Firm $A$ increases its quality level in response to the minimum quality standard ($\omega_A^\Omega > \omega_A$). In $F$, firm $C$ also increases the quality level of its product ($\omega_C^\Omega > \omega_C$).

Firm $A$ decreases its price in country $H$ ($p_A^\Omega, H < p_A, H$), firm $B$ increases its price ($p_B^\Omega, H > p_B, H$). This is, the price difference decreases. In country $F$, both firms increase their prices ($p_A^\Omega, F > p_A, F, p_C^\Omega, F > p_C, F$).

In $H$, the minimum quality standard shifts demand from the high quality firm $A$ to the low quality firm $B$ ($q_A^\Omega, H < q_A, H, q_B^\Omega, H > q_B, H$). In $F$, the minimum quality standard shifts demand from the high quality firm $C$ to the (relatively) low quality firm $A$ ($q_A^\Omega, F > q_A, F, q_C^\Omega, F < q_C, F$). This is, the volume of exports increases.

Proposition 4 Suppose that $\omega_A < \omega_C$ and that a mild minimum quality standard is introduced. In $H$, the minimum quality standard shifts demand from the high quality firm $A$ to the low quality firm $B$. In $F$, the minimum quality standard shifts demand from the high quality firm $C$ to the (relatively) low quality firm $A$. This is, the minimum quality standard increases the volume of exports.

2.2.3 Endogenous Minimum Quality Standard

Consider the introduction of an endogenous minimum quality standard in $H$, when the government sets the minimum quality standard to maximize welfare.

\(^5\)Since we are interested in the externalities of domestic minimum quality on foreign markets, we focus on one-way trade.
Autarky  Under autarky the government maximizes national welfare, the sum of consumer surplus and profits: \( W_H^* = CS_H^* + \pi_A^* + \pi_B^* \). This is equivalent to the case described in Ecchia & Lambertini (1997). Quality levels, prices, and quantities can be found in the Appendix.

Exports  Consider now the case of trade, where firm A exports to country F. If the government maximizes national welfare \( W_H = CS_H + \pi_A + \pi_B \), the minimum quality standard under trade is lower than under autarky (\( \Omega < \Omega^* \)). The quality level of firm A is also lower under trade than under autarky (\( \omega_A^\Omega < \omega_A^{\Omega^*} \)). Prices are also lower under trade than under autarky (\( p_A^{\Omega,H} < p_A^{\Omega^*,H}, p_B^{\Omega,H} < p_B^{\Omega^*,H} \)). Trade shifts demand from the low quality firm B to the high quality firm A (\( q_A^{\Omega,H} > q_A^{\Omega^*,H}, q_B^{\Omega,H} < q_B^{\Omega^*,H} \)).

Proposition 5  Suppose that \( \omega_A < \omega_C \) and that an endogenous minimum quality standard is introduced in country H. Then the welfare maximizing standard is lower under trade than under autarky.

2.3 Welfare

Consumer surplus, profits, and welfare can be found in the Appendix. The minimum quality standard increases consumer surplus in H (\( CS_H^\Omega > CS_H \)), decreases profits of firm A and firm B (\( \pi_A^\Omega < \pi_A, \pi_B^\Omega < \pi_B \)). In H, the minimum quality standard increases welfare if it is sufficiently low (\( W_H^\Omega > W_H \)). In F, the minimum quality standard increases consumer surplus (\( CS_F^\Omega > CS_F \)), but decreases profits of firm C (\( \pi_C^\Omega < \pi_C \)). The minimum quality standard reduces welfare in F (\( W_F^\Omega < W_F \)).

Proposition 6  Suppose that \( \omega_A < \omega_C \) and that an endogenous minimum quality standard is introduced in country H. In H, the minimum quality standard decreases profits of both firms. The minimum quality standard increases welfare. The minimum quality standard reduces welfare in F.

3 Conclusion

This paper has studied the interaction of a minimum quality standard and exports in a vertical product differentiation model where firms sell a global product. In particular, it has analyzed the effect of a national minimum quality standard on exports and the effect of exports on the choice of a national minimum quality standard.
The national minimum quality standard increases quality in the home market, but also in the export market, so it imposes an externality to foreign firms and foreign consumers. This externality should be taken into account by standard-setting authorities.

If ex-ante quality of foreign firms is lower (higher) than the quality of exporting firms, an exogenous mild minimum quality standard in the home market increases (decreases) the volume of exports. But the minimum quality standard also reduces profits of the exporting firm in both cases. So domestic minimum quality standards may be seen as an instrument to support exports in some cases, but this support is more in the interest of employees in exporting firms than in the interest the exporting firms themselves.

A domestic minimum quality standard increases welfare in the home country, but decreases welfare in the foreign country. A result of the product design policy of the EU might be an increased welfare within the EU at the expense of a lower welfare in other markets. This is, domestic standards may have a welfare-decreasing effect in some countries without imposing a barrier to trade. This effect might be an additional argument for an international coordination of standard-setting.

An endogenous welfare maximizing minimum quality standard is always lower under trade than under autarky. This is, trade has an influence on minimum quality standards. Lower domestic minimum quality standards are not only a prerequisite for international trade, it is the international trade that lowers the welfare-maximizing minimum quality standard. Against this background, the analysis of the harmonization of standards might be an interesting object of future research.
References


Appendix

Case 1: Low quality level in the export market

No Minimum Quality Standard

\[
\begin{align*}
\omega_A &= \frac{4b+1}{8} \\
\omega_B &= \frac{4b-5}{8} \\
\omega_C &= \frac{4b-5}{8} \\
P_{AH} &= \frac{8b+16b^2+25}{64} \\
P_{BH} &= \frac{-40b+16b^2+49}{64} \\
P_{AH} &= \frac{8b+16b^2+25}{64} \\
P_{CH} &= \frac{-40b+16b^2+49}{64} \\
q_{AH} &= \frac{1}{2} \\
q_{BH} &= \frac{1}{2} \\
q_{AH} &= \frac{1}{2} \\
q_{CH} &= \frac{1}{2} \\
\end{align*}
\]

Exogenous Minimum Quality Standard

\[
\begin{align*}
\omega_A^\Omega &= \frac{2(17b+20) - 9(17-\Omega) - 3\sqrt{4\Omega(17\Omega-17\Omega-20)} + 40b+17b^2+86}{59} \\
\omega_B^\Omega &= \Omega \\
\omega_C^\Omega &= \frac{31b-26-3\Omega - \sqrt{4\Omega(17\Omega-17\Omega-20)} + 40b+17b^2+86}{59} \\
P_{AH}^\Omega &= \frac{2(5263b+2312b^2+3554) + \Omega(4867b-6460b-6892) + 9\sqrt{4\Omega(17\Omega-17\Omega-20)} + 40b+17b^2+86}{(12\Omega-655-73)} \\
P_{BH}^\Omega &= \frac{-697b^2+4732b+7064+\Omega(7655b+27886-9464) + 27\sqrt{4\Omega(17\Omega-17\Omega-20)} + 40b+17b^2+86}{(2\Omega-22)} \\
P_{AH}^\Omega &= \frac{\Omega(1463b^2-3056b-3158)+8659b+3773b^2+9404+6\sqrt{4\Omega(17\Omega-17\Omega-20)} + 40b+17b^2+86}{(19\Omega-365-91)} \\
P_{BH}^\Omega &= \frac{\Omega(847\Omega-1378b-1996)+2(-1802b+1544b^2+5843) + 6\sqrt{4\Omega(17\Omega-17\Omega-20)} + 40b+17b^2+86}{(11\Omega-356-62)} \\
q_{AH}^\Omega &= \frac{2b+19-50(17-\Omega) + 3\sqrt{4\Omega(17\Omega-17\Omega-20)} + 40b+17b^2+86}{117} \\
q_{BH}^\Omega &= \frac{50\Omega-25b+158-3\sqrt{4\Omega(17\Omega-17\Omega-20)} + 40b+17b^2+86}{177} \\
q_{AH}^\Omega &= \frac{12b-6b+45+4\sqrt{4\Omega(17\Omega-17\Omega-20)} + 40b+17b^2+86}{177} \\
q_{BH}^\Omega &= \frac{2b+66-\Omega-2\sqrt{4\Omega(17\Omega-17\Omega-20)} + 40b+17b^2+86}{177} \\
\omega_A^\Omega - \omega_A &= \frac{3(-24\Omega+87+12b-8\sqrt{4\Omega(17\Omega-17\Omega-20)} + 40b+17b^2+86)}{472} > 0 \\
\omega_B^\Omega - \omega_B &= \Omega - \frac{4b-5}{8} > 0 \text{ by definition} \\
\omega_C^\Omega - \omega_C &= \frac{(24\Omega+87+12b-8\sqrt{4\Omega(17\Omega-17\Omega-20)} + 40b+17b^2+86)}{472} > 0 \\
P_{AH}^\Omega - P\_AH &= \frac{64\Omega(4867b-6460b-6892)+59012b+128846b^2+193837}{668352} \\
\end{align*}
\]
Endogenous Minimum Quality Standard

MQS under autarky (see Echca & Lambertini, 1997) \( \omega^* = \frac{20b + 3\sqrt{3} + 2}{40} \)

\[
\begin{align*}
\Omega^* &= \frac{20b + 9\sqrt{3} - 34}{40} \\
p^*_A &= \frac{b(20\sqrt{3} - 60) - 138\sqrt{3} + 200b^2 + 533}{800} \\
p^*_B &= \frac{b(180\sqrt{3} - 680) - 282\sqrt{3} + 200b^2 + 1037}{800} \\
q^*_A &= \frac{12 + 2\sqrt{\frac{5}{20}}}{20} \\
q^*_B &= \frac{8 + 2\sqrt{\frac{5}{20}}}{20}
\end{align*}
\]

MQS under trade, local \( W_H = CS_H + \pi_A + \pi_B \)

\( \omega_A - 2\omega_C - 2w_A\omega_C^2 + 2w_A\omega_C^2 + 8\omega_A - 4\omega_C - 8\omega_A^2 + 7\omega_C^2 + 4\omega_C^2 - 2\omega_C^2 - 14\omega_A^2 + 7b^2\omega_A + 4b_2^2 - 2b_2^2 \omega_C + \frac{\Omega(14b - 14\omega_A - 5\omega_A^2 - 5\omega_A^2 + 10\omega_A - 50\omega_A - 8)}{18} \)

\( \frac{dx_C}{d\omega_C} = \frac{-(b + \omega_A - 3\omega_C - 2)(b + \omega_A - 2\omega_C - 2)}{9} = 0 \rightarrow \omega_C^H(\omega_A) = \frac{(b + \omega_A - 2)}{3} \)

\( \frac{dW_H}{d\Omega} = \frac{-(-14b + 280\omega_A - 10\omega_A^2 + 10\omega_A^2 + 5\omega_A^2 + 8)}{18} = 0 \rightarrow \Omega(\omega_A) = \frac{10b - 5\omega_A + \sqrt{-70b + 25b^2 + 76 - 20\omega_A(5b - 5\omega_A - 7)}}{14} \)

\( \frac{dx_A}{d\omega_A} = \frac{4b - 8\omega_A - 8\omega_A^2 - 6\omega_A^2 - \omega_C^2 + 2b\omega_A + 2\omega_A\omega_C + 2b^2 + 2}{9} = 0, \)

\( \frac{dx_C}{d\omega_C} = 0 \rightarrow \omega_A^H(\Omega) = \frac{34b - 90\omega_A - 3\sqrt{40b + 17b^2 + 164 + 40(17b - 17\Omega + 20 + 40)}}{59} \frac{dx_A}{d\omega_A} = \frac{\left(\omega_A^*, \Omega(\omega_A), \omega_C^*(\omega_A)\right)}{11} = \frac{17}{225} - \frac{17}{350}\sqrt{6} < 0 \)

\( \frac{dW_H}{d\Omega} = \left(\omega_A^*(\Omega), \Omega^*, \omega_C^*(\omega_A)\right) \)
\[
\begin{align*}
&= (-\frac{51}{6962} \sqrt{6} + \frac{163}{3481}) \left(-\sqrt{\frac{4401}{100} \sqrt{6} + \frac{44757}{200}} - \frac{67545}{35268} \sqrt{6} + \frac{133936}{9067} \right) < 0 \\
\omega_A^* &< \omega_A^* \\
\Omega &< \Omega^*
\end{align*}
\]

**MQS under trade, global**

\[
W = CS_H + CS_F + \pi_A + \pi_B + \pi_C \\
= \frac{-2w_A - 8w_C - 5w_A^2 + 5w_A^2 + 5w_C^2 + 5w_C^2 + 80w_A - 10w_A^2 - 10w_C^2 + 10w_A^2 - 10w_C^2 - 50w_A^2 - 50w_C^2}{18} \\
+ \frac{(14b - 14\Omega - 5b^2 - 5\Omega^2 + 10\Omega - 5w_A^2 - 8)}{18} \\
dW = \frac{18}{18} \left( -14b + 28\Omega - 20\Omega^2 - 5w_A^2 + 10\Omega w_A + 5b^2 + 8 \right) \\
\Omega (\omega_A) = \frac{10b - 5w_A + \sqrt{-70 + 25b^2 + 76 - 20w_A(5b - 5w_A - 7)} - 14}{18} \\
as above, \\
\omega_A^* &< \omega_A^* \\
\Omega &< \Omega^*
\]

**Case 2: High quality level in the export market**

\[
\omega_A < \omega_C
\]

**No Minimum Quality Standard**

\[
\omega_A = \frac{2b - 1}{4} \\
\omega_B = \frac{2b - 3}{4} \\
\omega_C = \frac{2b + 1}{4} \\
p_{A,H} = \frac{-12b + 12b^2 + 19}{48} \\
p_{B,H} = \frac{-36b + 12b^2 + 35}{48} \\
p_{A,F} = \frac{-12b + 12b^2 + 19}{48} \\
p_{C,F} = \frac{12b + 12b^2 + 11}{48} \\
q_{A,H} = \frac{2}{3} \\
q_{B,H} = \frac{1}{3} \\
q_{C,F} = \frac{1}{3} \\
q_{A,F} = \frac{2}{3}
\]

**Exogenous Minimum Quality Standard**

\[
\omega_A^* = \frac{9b - 2(b + 28) + 3\sqrt{b(b + 56) + 334 - 4\Omega(b - \Omega + 28)}}{5} \\
\omega_B^* = \Omega \\
\omega_C^* = \frac{b - 17 + 3\Omega + \sqrt{b(b + 56) + 334 - 4\Omega(b - \Omega + 28)}}{5}
\]
\[ P_{A,H}^\Omega = \frac{\Omega(25971-124b-4012)+2\sqrt{5836+8b^2+6002}+9\sqrt{b(b+56)+334-4\Omega(6-\Omega+28)(12\Omega-b-73)}}{75} \]

\[ P_{B,H}^\Omega = \frac{\Omega(1671-92b-1796)+986b+23b^2+5582+9\sqrt{b(b+56)+334-4\Omega(6-\Omega+28)(6\Omega-3b-34)}}{75} \]

\[ P_{A,F}^\Omega = \frac{\Omega(247\Omega-112b+4306)+131\Omega b+131b^2+13297+6\sqrt{b(b+56)+334-4\Omega(6-\Omega+28)(19\Omega-2b-121)}}{75} \]

\[ P_{C,F}^\Omega = \frac{\Omega(143\Omega-98b-2474)+982b+32b^2+7583+6\sqrt{b(b+56)+334-4\Omega(6-\Omega+28)(11\Omega-3b-69)}}{75} \]

\[ q_{A,H}^\Omega = \frac{(7b-14\Omega+61-3\sqrt{b(b+56)+334-4\Omega(6-\Omega+28)})}{15} \]

\[ q_{B,H}^\Omega = \frac{14\Omega-7b-46+3\sqrt{b(b+56)+334-4\Omega(6-\Omega+28)}}{15} \]

\[ q_{C,F}^\Omega = \frac{(3b+39-6\Omega-2\sqrt{b(b+56)+334-4\Omega(6-\Omega+28)})}{15} \]

\[ q_{A,F}^\Omega = \frac{12\Omega-63-6b+4\sqrt{b(b+56)+334-4\Omega(6-\Omega+28)}}{15} \]

\[ \omega_{A}^\Omega - \omega_{A} = \frac{3(12\Omega-6b-73+4\sqrt{b(b+56)+334-4\Omega(6-\Omega+28)})}{20} > 0 \]

\[ \omega_{B}^\Omega - \omega_{B} = \Omega - \frac{2b-3}{4} > 0 \]

\[ \omega_{C}^\Omega - \omega_{C} = \frac{(12\Omega-6b-73+4\sqrt{b(b+56)+334-4\Omega(6-\Omega+28)})}{20} > 0 \]

\[ P_{A,H}^\Omega - P_{A,H} = \frac{16\Omega(25971-124b-4012)-44b^2+18956b+191589}{1200} < 0 \]

\[ + \frac{144\sqrt{b(b+56)+334-4\Omega(6-\Omega+28)}(12\Omega-b-73)}{1200} \]

\[ P_{B,H}^\Omega - P_{B,H} = \frac{16\Omega(1671-92b-1796)+16708b+688b^2+88437}{1200} \]

\[ + \frac{144\sqrt{112b+56b-41b^2+b^2+334(6\Omega-3b-34)}}{1200} \]

\[ P_{A,F}^\Omega - P_{A,F} = \frac{16\Omega(247\Omega-112b+4306)-92b^2+21308b+212277}{1200} \]

\[ + \frac{96\sqrt{b(b+56)+334-4\Omega(6-\Omega+28)}(19\Omega-2b-121)}{1200} \]

\[ > 0 \]

\[ P_{C,F}^\Omega - P_{C,F} = \frac{16\Omega(143\Omega-98b-2474)+15412b+2122b^2+121053}{1200} \]

\[ + \frac{96\sqrt{b(b+56)+334-4\Omega(6-\Omega+28)(11\Omega-3b-69)}}{1200} \]

\[ > 0 \]

\[ q_{A,H}^\Omega - q_{A,H} = -\frac{14\Omega-7b-51-3\sqrt{b(b+56)+334-4\Omega(6-\Omega+28)}}{15} < 0 \]

\[ q_{B,H}^\Omega - q_{B,H} = \frac{14\Omega-7b-51+3\sqrt{b(b+56)+334-4\Omega(6-\Omega+28)}}{15} > 0 \]

\[ q_{C,F}^\Omega - q_{C,F} = -\frac{(12\Omega-6b+4\sqrt{b(b+56)+334-4\Omega(6-\Omega+28)}-73}{15} < 0 \]

\[ q_{A,F}^\Omega - q_{A,F} = \frac{(12\Omega-6b+4\sqrt{b(b+56)+334-4\Omega(6-\Omega+28)}-73}{15} > 0 \]

Endogeneous Minimum Quality Standard

MQS under trade, local

\[ W_H = \frac{-9\omega_{A}+8\omega_{C}-6b\omega_{A}+36\omega_{A}^2+26\omega_{C}+2\omega_{A}\omega_{C}-2\omega_{A}^2\omega_{C}-12\omega_{A}^2+3\omega_{A}^3+8\omega_{C}^2+2\omega_{C}^3+12b\omega_{A}-8b\omega_{C}}{18} + \Omega\left(14b-14\Omega-5b^2+5b^2\Omega+10b\Omega-5\Omega\omega_{A}-8\right) \]

\[ \omega_{A} = \frac{7b+12\sqrt{7}-35}{14} \]

\[ \Omega = \frac{7b-10\sqrt{7}+21}{14} \]

16
\[
\begin{align*}
\omega_C &= \frac{7b+4\sqrt{7}-7}{14} \\
p_{A,H}^\Omega &= \frac{b(72\sqrt{7}-210)+21b^2-256\sqrt{7}+689}{84} \\
p_{B,H}^\Omega &= \frac{-b(60\sqrt{7}-126)+21b^2-152\sqrt{7}+421}{84} \\
p_{A,F}^\Omega &= \frac{b(72\sqrt{7}-210)+21b^2-280\sqrt{7}+773}{84} \\
p_{C,F}^\Omega &= \frac{b(24\sqrt{7}-42)+21b^2-152\sqrt{7}+421}{84} \\
q_{A,H}^\Omega &= \frac{14-\sqrt{7}}{21} \\
q_{B,H}^\Omega &= \frac{\sqrt{7}+7}{21} \\
q_{A,F}^\Omega &= \frac{8\sqrt{7}-7}{21} \\
q_{C,F}^\Omega &= \frac{28-8\sqrt{7}}{21} \\
\omega_A^*-\omega_A^* &= \frac{-21\sqrt{7}+240\sqrt{7}-714}{280} < 0 \\
\Omega - \Omega^* &= \frac{-63\sqrt{7}-200\sqrt{7}+658}{0} < 0 \\
p_{A,H}^\Omega - p_A^* &= \frac{-b(280+1260\sqrt{7}-14-1400\sqrt{7})-2898\sqrt{7}+51 200\sqrt{7}-126 607}{16800} < 0 \\
p_{B,H}^\Omega - p_B^* &= \frac{-b(3780\sqrt{7}+12 000\sqrt{7}-39 480)-5922\sqrt{7}+30 400\sqrt{7}-62 423}{16800} < 0 \\
q_{A,H}^\Omega - q_A^* &= \frac{-21\sqrt{7}+10\sqrt{7}+14}{210} > 0 \\
q_{B,H}^\Omega - q_B^* &= \frac{-21\sqrt{7}+10\sqrt{7}+14}{210} < 0 \\
\omega_A^* - \omega_A &= \frac{24\sqrt{7}-63}{28} > 0 \\
\omega_B^* - \omega_B &= \frac{63-20\sqrt{7}}{28} > 0 \\
\omega_C^* - \omega_C &= \frac{8\sqrt{7}-21}{28} > 0 \\
p_{A,H}^\Omega - p_{A,H} &= \frac{-b(280+1260\sqrt{7}-14-1400\sqrt{7})-2898\sqrt{7}+51 200\sqrt{7}-126 607}{288\sqrt{7}-756} < 0, \text{ if } b < \frac{(1024\sqrt{7}-2623)}{288\sqrt{7}-756} \\
p_{B,H}^\Omega - p_{B,H} &= \frac{b(756-288\sqrt{7})+1024\sqrt{7}-2623}{336} > 0 \\
p_{A,F}^\Omega - p_{A,F} &= \frac{b(756-240\sqrt{7})-608\sqrt{7}+1439}{336} > 0 \\
p_{C,F}^\Omega - p_{C,F} &= \frac{b(96\sqrt{7}-252)-608\sqrt{7}+1607}{336} > 0 \\
q_{A,H}^\Omega - q_{A,H} &= \frac{-1}{21}\sqrt{7} < 0 \\
q_{B,H}^\Omega - q_{B,H} &= \frac{8\sqrt{7}-21}{21} > 0 \\
q_{A,F}^\Omega - q_{A,F} &= \frac{8\sqrt{7}-21}{21} > 0 \\
q_{C,F}^\Omega - q_{C,F} &= \frac{-8\sqrt{7}-21}{21} < 0 \\
\end{align*}
\]

**MQS under trade, global**

\[
W = \frac{188\omega_A+5b^2\omega_C-10b\omega_C^2+4b\omega_C-5b\omega_C^2-18\omega_C^2+5\omega_A^2\omega_C^2-9\omega_A+5\omega_C^2-4\omega_C^2-\omega_C}{18} + \frac{14\omega_C}{18}\Omega(14b-14\Omega-5b^2+5\omega_C^2+10b\omega_C^2-5b\omega_A-8) \\
\omega_A = \frac{7b+12\sqrt{7}-35}{14} \quad \Omega = \frac{7b-10\sqrt{7}+21}{14} \quad \omega_C = \frac{7b+4\sqrt{7}-7}{14}
\]
3.1 Welfare Analysis

3.1.1 Case 1: Low quality level in the export market

No Minimum Quality Standard  \( CS = \left( \frac{1}{4} b^2 - \frac{1}{4} b - \frac{23}{64} \right) \)

\[ \pi_A = \frac{3}{8}, \quad \pi_B = \frac{3}{16} \]

\[ W_H = \frac{1}{4} b^2 - \frac{1}{4} b + \frac{13}{64} \]

\[ CS_F = \frac{1}{4} b^2 - \frac{1}{4} b - \frac{23}{64} \]

\[ \pi_C = \frac{3}{16} \]

\[ W_F = \frac{1}{4} b^2 - \frac{1}{4} b - \frac{11}{64} \]

Exogenous Minimum Quality Standard  \( CS_H = \frac{498 
\text{2026}^4 + 9781 
\text{312}^3 - 57 
\text{055} 
\text{341}^2 - 179 
\text{672} 
\text{264} - \text{b} - 154 
\text{360} 
\text{718} 
\text{3696} 
\text{822} 
\text{34b} - 68 \Omega + 40 - 3 \sqrt{4 \Omega (17 \Omega - 17 \Omega - 20)} \text{b} + 40 \text{b} + 17 \text{b}^2 + 86 \]

\[ \pi_A = \frac{3}{8}, \quad \pi_B = \frac{3}{16} \]

\[ W_H = \frac{1}{4} b^2 - \frac{1}{4} b + \frac{13}{64} \]

\[ CS_F = \frac{1}{4} b^2 - \frac{1}{4} b - \frac{23}{64} \]

\[ \pi_C = \frac{3}{16} \]

\[ W_F = \frac{1}{4} b^2 - \frac{1}{4} b - \frac{11}{64} \]
\[ 3 \sqrt{4(17\Omega - 17b - 20) + 40b + 17b^2 + 86} \left( 41( -34 495b + 34 495\Omega - 73926b + 4928b^2 + 3696b^3 + 327 997) \right) \]
\[ \frac{3696 822}{3666 - 601 - 2 \sqrt{4(17\Omega - 17b - 20) + 40b + 17b^2 + 86}} \]
\[ \pi_C = 2556b^3 - 15 336b^2 + 66 744b + 30 627b^2 - 266 976b^2 + 295 920b - 20 440\Omega^3 + 266 976b^2 - 591 840\Omega + 1422 432 \]
\[ 8 \sqrt{4(17\Omega - 17b - 20) + 40b + 17b^2 + 86} \left( 95b^3 - 380b^2 + 1348b + 380b^2 - 2696b^2 + 13 412 \right) \]
\[ W_F = -245 464b - 125 209b - 94 342\Omega^3 + 94 342b^2 - 20 440\Omega^3 + 30 672b^2 - 15 336b^2 + 331 654b^2 + 2556b^3 - 104 960 \]
\[ + \sqrt{4(17\Omega - 17b - 20) + 40b + 17b^2 + 86} \left( -4058b + 81 166b + 30 40b^3 + 30 40b^2 - 760b^2 - 11 539 \right) \frac{1848 411}{1232 274} \]
\[ CS_{H}^{\Omega} - CS_{H} = - \frac{22 794 000b - 20 019 888b^2 + 601 664b^3 - 37 335 763}{118 298 304} \]
\[ - \frac{32\Omega(-250 2 486b + 250 2 486b + 150 416b^2 + 112 812b^2 - 1424 625)}{118 298 304} \]
\[ + \frac{96 \sqrt{4(17\Omega - 17b - 20) + 40b + 17b^2 + 86}(5168b - 10 336b^2 - 3688b^3 + 3688b^2 + 922b^2 + 70 477)}{118 298 304} \]
\[ \pi_A - \pi_A = 4728b + 8688b^2 + 28 96b^3 - 69 779 \]
\[ = 16\Omega(217b - 217b - 217b - 1440\Omega^2 + 1086b^2 + 591) \]
\[ - \frac{250 632}{29 744 576} \]
\[ + \frac{8 \sqrt{4(17\Omega - 17b - 20) + 40b + 17b^2 + 86}(62b - 12 14\Omega - 136b^2 + 136b^2 + 346b^3 + 307)}{29 744 576} \frac{10 376 832b - 31 118 72b^2 + 30 832b^3 + 14 840 431}{29 744 576} \]
\[ + \frac{64\Omega(-194 892b + 3 406b + 36 04b^2 + 26 203b^2 + 324 276)}{29 744 576} \]
\[ + \frac{8 \sqrt{4(17\Omega - 17b - 20) + 40b + 17b^2 + 86}(62b - 12 14\Omega - 136b^2 + 136b^2 + 346b^3 + 307)}{29 744 576} \frac{10 376 832b - 31 118 72b^2 + 30 832b^3 + 14 840 431}{29 744 576} \]
\[ \frac{96 \sqrt{4(17\Omega - 17b - 20) + 40b + 17b^2 + 86}(62b - 12 14\Omega - 136b^2 + 136b^2 + 346b^3 + 307)}{29 744 576} \]
\[ > 0, \text{ if } \Omega < \Omega^* \]

for \( b = 2, \Omega^* = 0.436 45 \)

for \( b = 5, \Omega^* \approx 2.187 \)

for \( b = 10, \Omega^* \approx 4.686 \)

\[ W_H - W_H = 20 944 944b - 28 392 240b^2 + 31 719 904b - 10 909 727 \]
\[ \frac{118 298 304}{118 298 304} \]
\[ - 32\Omega(-3549 036b + 3549 036b - 1189 464b + 792 976b^2 + 594 732b^2 + 1309 059) \]
\[ + \frac{32 \sqrt{4(17\Omega - 17b - 20) + 40b + 17b^2 + 86}(15 596b - 31 192b - 49 240b + 49 240b^2 + 12 310b^2 + 17 389)}{118 298 304} \]
\[ > 0, \text{ if } \Omega < \Omega^* \]

for \( b = 2, \Omega^* = 0.6105 \)

for \( b = 5, \Omega^* \approx 2.36 \)

for \( b = 10, \Omega^* \approx 4.861 \)

\[ CS_{F}^{\Omega} - CS_{F} = 3 \left( -4309 616b - 669 136b^2 + 27 264b^3 - 19 532 785 \right) \]
\[ \frac{118 298 304}{118 298 304} \]
\[ - 96\Omega(-83 642b + 83 642b - 10 224b + 68 16b^2 + 51 120b^2 - 269 351) \]
\[ > 0, \text{ if } \Omega < \Omega^* \]

for \( b = 2, \Omega^* = 0.6105 \)

for \( b = 5, \Omega^* \approx 2.36 \)

for \( b = 10, \Omega^* \approx 4.861 \)
3.1.2 Case 2: High quality level in the export market

No Minimum Quality Standard \[ CS_H = \frac{1}{4}b^2 - \frac{1}{4}b - \frac{35}{16} \]

\[ \pi_A = \frac{4}{9} \]
\[ \pi_B = \frac{1}{18} \]
\[ W_H = \frac{1}{4}b^2 - \frac{1}{4}b + \frac{37}{16} \]
\[ CS_F = \frac{1}{4}b^2 - \frac{1}{4}b - \frac{35}{16} \]
\[ \pi_C = \frac{1}{18} \]
\[ W_F = \frac{1}{4}b^2 - \frac{1}{4}b - \frac{3}{16} \]

Exogenous Minimum Quality Standard \[ CS_H = \frac{30 653 704b + 30 21 387b^2 + 110 416b^3 + 125 884b^4 + 100 967 338}{6750b(b+56)+334-42b(b+28)-4500b+90 000b^2-126 000} \]

\[ \pi_A = \frac{5}{1125} \left( -24 936 - 150b^2 + 2b^3 - 8713 \right) \]
\[ \pi_B = \frac{1}{1125} \left( -30 000 + 300b^2 - 1290 + 8b^3 + 8b^2 - 2439 \right) \]
\[ \pi_C = \frac{6}{1125} \left( 286 536b + 21 582b^2 + 353b^3 + 1062 401 \right) \]
\[ W_H = \frac{174 906b - 174 906b^2 - 84 726b + 5648b^2 + 4236b^2 + 1145 019}{2250} \]
\[ CS_F = \frac{2 (6877 433b + 894 05b^2 + 43 787b^3 + 6 26b^4 + 17 999 231)}{2250} \]
\[ \Omega = (7068 057b + 7071 432b^2 + 1030 368b^3 - 687 092b^4 + 20 632b^5 - 40 040b^6 + 30 048b^7 - 5 186 694b^8 - 10 016b^9 - 27 553 607) \]

\[ \frac{3 200 b^{10} b^{10} + 334 - 140 b (b + 28) + 7 545 b + 1 749 b^2}{6750 b^{10} b^{10} - 4500 b^{10} b^{10} + 334 - 42 b (b + 28)} \]
\[
\pi_C = 4\left(51.921b^3 + 35.372b^2 + 639b + 215.631\right)
- \frac{4(18\Omega(786b - 786\Omega - 42\Omega + 28\Omega^2 + 215.631 + 5769))}{1125}
- 4\left(2\sqrt{b(56) + 334 - 4\Omega(b - \Omega + 28)(926b + 315^2 + 5899 - 4\Omega(31b - 31\Omega + 463))}\right)
\]
\[
W_F = \frac{1125}{750}\left(3b - 6\Omega - 2\sqrt{b(56) + 334 - 4\Omega(b - \Omega + 28) + 39}\right)
+ \frac{\Omega(-8394.867b + 8395.992\Omega + 1102.568\Omega - 734.972b^2 + 20.032\Omega^3 + 40.064\Omega^2 + 30.048\Omega^2(552.354b^2 - 10.016b^3 - 37.079.597))}{750}
+ \frac{\sqrt{b(56) + 334 - 4\Omega(b - \Omega + 28)(2656 - 3312\Omega + 344b^2 + 866b^2 + 16619) - 0}{225}
- \frac{\sqrt{b(56) + 334 - 4\Omega(56 - 28)(94b - 188\Omega - 86\Omega + 8\Omega^2 + 2b^2 + 479))}{225}
< 0
\]
\[
\pi_A - \pi_A = -24.399 + 150b^2 - 2b^3 + 8813
- \frac{24\sqrt{b(56) + 334 - 4\Omega(b - \Omega + 28)(645b^2 + 150\Omega + 8\Omega^2 + 66b^2 + 2439)}{18000}
+ \frac{\sqrt{b(56) + 334 - 4\Omega(b - \Omega + 28)(126b^2 + 26.856b^2 + 484b^3 + 1126b^3 + 81.492)}{22500}
+ \frac{12\sqrt{b(56) + 334 - 4\Omega(b - \Omega + 28)(1058 - 2116b - 172b^2 + 43^2 + 5137))}{22500}
< 0
\]
\[
W_H - W_H = \frac{-4580.076b^3 - 349.812b^3 - 56.48b^3 - 17.003b^4}{18000}
+ \frac{8\sqrt{b(56) + 334 - 4\Omega(b - \Omega + 28)(216b^2 - 43.08b^2 + 376b^2 + 376b^2 + 794b^2 + 116.29))}{18000}
> 0
\]
if \(\Omega < \Omega^*\)

for \(b=2, \Omega^* = 0.645 82\)

for \(b=5, \Omega^* \approx 2.396\)

for \(b=10, \Omega^* \approx 4.896\)

\[
CS_H^B - CS_H^F = \frac{1152.852b + 98.364b^2 + 2016b^3 + 3814.087}{18000}
- \frac{24\Omega(16.394b - 16.394b + 10.08b + 672b^2 + 504b^2 + 96.071)}{18000}
- \frac{8\sqrt{b(56) + 334 - 4\Omega(b - \Omega + 28)(569b^2 - 11.396b^2 - 992b^2 + 248b^2 + 26.087)}}{18000}
> 0
\]
\[
\pi_C - \pi_C = \frac{415.368b + 28.02b^2 + 504b^3 + 1724.923}{22500}
- \frac{144\Omega(786b - 786b - 42b^2 + 28b^2 + 21b^2 + 57b^2)}{22500}
- \frac{16\sqrt{b(56) + 334 - 4\Omega(b - \Omega + 28)(926b + 1852b^2 - 1248b^2 + 124b^2 + 31b^2 + 5899))}{22500}
< 0
\]
\[
W^\Omega - W_F = -\frac{1491.9326 - 108.244 b^2 - 2016b^3 - 5871.157}{80 (54.122b - 30.244b + 2016b^3 + 1512b^2 + 372.983)} - 8\sqrt{b(b+65)+334-4\Omega(b+28)}\frac{(6835 - 13.676b - 9924b^2 + 248b^2 + 40.157)}{6000} < 0
\]

Endogeneous Minimum Quality Standard

\[
CS_H^\Omega = \frac{1}{4}b^2 - \frac{1}{4}b + \frac{1171}{441}\sqrt{7} - \frac{443}{63}
\]

\[
\pi_A^\Omega = \frac{46}{63} - \frac{11}{63}\sqrt{7}
\]

\[
\pi_B^\Omega = \frac{32}{63}\sqrt{7} - \frac{10}{63}
\]

\[
W_H^\Omega = \frac{1}{4}b^2 - \frac{1}{4}b + \frac{1126}{441}\sqrt{7} - \frac{407}{63}
\]

\[
CS_H^\Omega = \frac{1}{4}b^2 - \frac{1}{4}b + \frac{481}{441}\sqrt{7} - \frac{197}{63}
\]

\[
\pi_C = \frac{608}{63} - \frac{1600}{441}\sqrt{7}
\]

\[
W_F^\Omega = \frac{1}{4}b^2 - \frac{1}{4}b - \frac{373}{441}\sqrt{7} + \frac{137}{21}
\]

\[
CS_H^\Omega - CS_H = \frac{1171}{441}\sqrt{7} - \frac{2281}{336} > 0
\]

\[
\pi_A - \pi_A = \frac{2}{7} - \frac{11}{63}\sqrt{7} < 0
\]

\[
\pi_B - \pi_B = \frac{32}{441}\sqrt{7} - \frac{3}{44} < 0
\]

\[
W_H^\Omega - W_H = \frac{1126}{441}\sqrt{7} - \frac{2281}{336} > 0
\]

\[
CS_F^\Omega - CS_F = \frac{481}{441}\sqrt{7} - \frac{323}{112} > 0
\]

\[
\pi_C - \pi_C = \frac{403}{42} - \frac{1600}{441}\sqrt{7} < 0
\]

\[
W_F^\Omega - W_F = \frac{2255}{336} - \frac{373}{441}\sqrt{7} < 0
\]