Legal change in the face of uncertainty

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Abstract

This study investigates the optimal nature of legal change under uncertainty. I focus on a case in which a harmful activity will be subjected to some regulatory measures (a standard, exposure to liability, or a corrective tax). As the benefits and costs of precaution are ex-ante uncertain, so are the precise contours of the measures to be taken. This situation places an uncertainty burden on both injurers and victims. The optimal policy should, at the same time, strike a balance between benefits and costs of the measures, and attenuate this uncertainty burden. Whether measures should be made stronger or softer depends on the size and the sign of the shocks affecting the parties (positive or negative) and their disposition towards risk. With corrective taxes, it also depends on the elasticity of precautions with respect to the tax rate.

Keywords: legal change; regulation of risk; externalities; cost-benefit analysis; emerging risks.

JEL codes: K2 (Regulation and Business Law), D62 (Externalities), L51 (Economics of regulation)

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This search for static security - in the law and elsewhere - is misguided. The fact is that security can only be achieved through constant change …"  

1 Introduction

Just as politics, human knowledge, and technology vary in time, so does the law. Not infrequently, the new rules promulgated by legislatures, courts, and regulators turn out to impose very high costs on the parties affected.\(^1\) Consider the case in which a new regulation outlaws a widely used technology. The replacement of this technology with a new one (say, a "cleaner" one) imposes substantial costs on people who were relying on the old technology, regarded as fully lawful until the time of the change. Should the parties negatively affected by this change be compensated for the loss suffered? In other terms, should the government provide affected parties with insurance against its own decisions? If so, how can this insurance best be provided?

The issue of the compensation of those who stand to lose from a policy change has long been the crux of law and economics scholarship. The early literature, building on the contributions of Michelman (1967) and Feldstein (1976), argued that losses caused by unilateral acts of a policy maker (such as a property or a regulatory taking) have to be compensated, in order to not frustrate legitimate expectations and to promote reliance on the law.\(^2\) This perspective has been challenged by the so-called "new view," notably advanced by Graetz (1977) and Kaplow (1986). Both authors note that the risks of legal change are not substantially different from the risks created by the market (due, for example, to changes in technology and tastes). As the market does not reward wrong expectations, the government should not pay tribute to expectations that might turn out to be wrong (such as the expectation that the law is immutable). Even if

\(^1\)From 2006 through 2016, for example, the EPA and DOT alone have promulgated 67 major rules whose total estimated annual costs lie in the range of 73.3 to 100.4 billion dollars (see OMB (2017)).

\(^2\)Failure to keep the current law would produce "demoralization costs" for Michelman and "inefficient precautionary behavior" for Feldstein. Furthermore, legal changes with arbitrary distributive effects - and no compensation - would breach "horizontal equity" (Feldstein (1976)).
expectations are correct on average, however, and parties are aware that the law might change, no insurance should be provided against the occurrence of the worse outcomes (such that the costs associated with the new law turn out to be the highest among the prospected ones). As forcefully argued by Kaplow, there is no reason to presume that the government is in a better position to provide insurance against these risks than the private market. As no government insurance is provided against adverse market changes, no transition relief should be granted to those who are adversely affected by legal change. The new view is postulated on the premise that policy making is the outcome of proper balancing of benefits and costs of change, so the costs borne by the parties are part and parcel of a socially desirable legal change.\(^3\)

While Kaplow’s argument that insurance should be provided by the private market is theoretically convincing, it runs against a major practical obstacle: insurance policies do not normally cover losses resulting from legal change. Legal change happens to be a largely uninsurable risk (Shavell (2014b)). This puzzling fact has attracted recurrent scrutiny. Several explanations have been provided, including moral hazard and adverse selection (Blume and Rubinfeld (1984)) and pricing difficulties (Masur and Nash (2010)). Shavell (2014b) highlights the correlated nature of the costs borne by the parties to comply with the new law. When a new environmental regulation outlaws a production technology or a work safety regulation imposes new prevention measures, a myriad of parties are affected in the same way at the same time. If insurance companies pooled these risks together, their outlays would be subject to extreme volatility. This makes private insurance very costly - though not necessarily infeasible.\(^4\)

Given that private insurance is not available, transition relief becomes an attractive option. For this purpose, a whole set of policy instruments are regularly employed, both

\(^3\)The latter proposition is in line with the standard efficiency-based approach of law and economics, which follows cost-benefit analysis in positing that a policy move is desirable when compensation of losing parties is just "feasible." Compensation itself is not required, as it pertains to the realm of "equity" (and is, thus, left to the tax system).

\(^4\)Insurance policies cover at most liability for harm caused. Legal development risk is likely to create negatively correlated losses in this field too (Baker (2004)). This has put pressure on the market (especially with respect to medical liability insurance), but has not disrupted it.
temporary and permanent, including specific exemptions, waivers, subsidies, grandfathering, and phase-ins.\(^5\)

In this paper, I build on the contribution of Shavell (2014b). In his insightful analysis, Steven Shavell argues for a special kind of relief for parties negatively affected by legal change: attenuation. He shows that the risk borne by the parties who have to comply with the new law or regulation can be mitigated by making the new regulation less stringent: if the perspective compliance costs decline, so does the uncertainty burden attendant with them. In his discussion - and following most of the literature - Shavell emphasizes the cost side of the new regulation, thereby arguing that uncertainty calls for attenuation of legal change.\(^6\)

I extend Shavell’s contribution and develop a broader picture of legal change, in which parties can be affected both in a negative and a positive way. I consider a case in which, at a later date, a measure will be enacted to curb the harmful effects of an activity. Both the costs and the benefit side of this measure are subject to ex-ante uncertainty: the costs of precaution for the injurers depend on potential technological innovation, the benefits of precaution (the reduction in the probability of harm) depend on outside factors (such as evolving ecological conditions or better scientific understanding of the harm). The different combinations of harm and costs create potential upside and downside risks for injurers and potential victims. In order to reduce the ex-ante uncertainty burden of the parties, the policy maker should smooth out extreme outcomes, for instance by requesting less precaution when compliance costs are particularly high, or by requesting greater precaution when the risk of harm is particularly high. Therefore, depending on the circumstances, regulatory measures should be tightened

\(^5\)See, among others, Levmore (1999), Shaviro (2000), Nash and Revesz (2007), Masur and Nash (2010), and Shavell (2014a). These authors defend transition relief on the basis of a variety of arguments, including efficiency, incentives for socially desirable investments, governmental legitimacy, and fairness. From a public choice perspective, compensation for takings has been defended as a means to avoid "fiscal illusion" by the government, i.e. the illusion that takings come at no social cost because they do not trigger a monetary outlay (see, among others, Blume and Rubinfeld (1984) and Miceli and Segerson (1994)).

\(^6\)Kaplow advances an informal version of the attenuation principle when he argues that by "scaling down" a reform that balances benefits and costs, ex-ante risk can be reduced (Kaplow (1986), pp. 588-89). However, he dismisses this option on the premise that private insurance is preferable.
or relaxed.

The direction in which the regulatory measures should be changed depends on the sign of the shocks parties are subject to, the size of these shocks, and the parties’ disposition towards risk. Independently of the number of injurers and victims, the relevant factor is the size of individual shocks (and not aggregate shocks).

A remarkable feature of my results is that the reduction of ex-ante uncertainty is welfare improving also when uncertainty is about harm’s probability, rather than its magnitude. Thus, as I explain in Section 2, the attenuation policy should treat parties as if they displayed aversion to compound lotteries.\footnote{Compound lotteries are lotteries whose prizes are lottery tickets. For a rational agent, all that matters is the compound probability of winning: "... only algebra, not human behavior, is involved in this definition" (Samuelson (1952)).}

Following Shavell (2014b), I study three policy tools: 1) a \textit{regulation} imposing a standard of behavior to limit harmful effects (this could be a command and control limit on emissions, a technology standard, or a negligence standard), 2) \textit{liability} for the harm caused (a policy that enables victims to claim damages for the harm suffered), and 3) \textit{corrective taxation}, i.e., taxes or subsidies encouraging the adoption of preventive measures.

With respect to regulation, I show that the standard should be made tighter if: i) the injurers are subject to a negative shock and the victims to a positive shock, ii) both injurers and victims are subject to a negative shock and victims are better able to take additional risk, iii) both injurers and victims are subject to a positive shock, and injurers are in a better position to take additional risk. A similar result applies to liability for harm. Damages should be reduced (with respect to the otherwise optimal level) if conditions equivalent to those listed above apply. I also show that, in contrast to the classic result of Shavell (1982), here, optimal damages can exceed harm (when the injurer is subject to a positive shock).

The results for the taxation case are notably different.\footnote{In Shavell (2014b), liability and corrective taxes are equivalent because victims are assumed to be indifferent to risk (so, whether they are compensated or not makes no difference, from an efficiency perspective).} Here, an increase in the marginal tax rate affects the payoff of the injurers by means of variations in their net
outlays (an increase if injurers are taxed, a decrease if injurers are subsidized). The increase in the marginal rate affects the victims through the reduction in harm brought about by the additional precautions exerted by the injurers. If precautions are very sensitive to the tax rate, the latter effect will dominate.

The attenuation logic requires that the outcome of traditional cost-benefit analysis be modified to mitigate the ex-ante risk borne by the parties. Mitigation is more important when parties are subject to great shocks (negative and positive). From a policy perspective, this implies that cost-benefit analysis should include some "stabilization" corrections, aimed at avoiding excessive costs for adversely affected parties. Such corrections have been recently advocated in environmental policy, where specific measures have been proposed (safety valves, allowance banking, and collars) to limit the payoff volatility associated with emission control mechanisms.\(^9\) These measures exclusively focus on compliance costs. My results keep both sides of the "market" into account, and highlight the value also of stabilization measures aimed at curbing the volatility of the victims’ payoff.

In contrast to much of the literature on legal change that focuses on the uncertainty about the policy measure itself, my paper focuses on the fundamental uncertainty that triggers the policy decision (uncertainty about the costs and benefits of the measure). From this perspective, "inaction" can create greater risk for society than "action" (whence the opening quote). Newfound harms or new terrorist threats represent negative shocks that should be mitigated by means of a tough policy stance - to an extent that goes beyond momentary balancing of benefits and costs.\(^{10}\) In this sense, this paper provides some justification to the over-reaction that normally follows public scares, in the vicious circle at the core of Justice Breyer’s concerns (see Breyer (2009)).

The paper proceeds as follows. In Section 2, I illustrate the logic of risk attenuation. The basic factors guiding the optimal attenuation policy are presented. In Section 3, I develop a formal model in which both the cost of precaution and the probability of

\(^9\)See, for instance, Aldy and Viscusi (2014), Aldy (2017), and references therein.

\(^{10}\)Incidentally, the same logic applies to emergency rescues: the ex-ante value of mitigating the risk for the victims of adverse events exceeds the ex-post benefits accruing to the individual victims.
harm are subject to ex-ante uncertainty. The policy maker decides the optimal standard of behavior to be followed after uncertainty has unraveled. In Section 4, I consider a case in which potential injurers face liability for the harm caused. The policy maker decides the level of damages to be awarded to the victims (these damages can under- or over-compensate victims). Section 5 deals with corrective taxation. The tax, which can be negative (i.e., a subsidy), decreases with the level of precaution taken. It shares the features of a standard emission fee (such as a carbon fee). Section 7 concludes.

2 The attenuation principle

Let us consider a simple illustration of Shavell’s attenuation principle. Consider the case of a potentially harmful activity that will be subject to regulation in the near future. At the moment, we do not know with certainty what precautionary measures will be mandated, because the cost of precaution and the probability of harm are uncertain.\(^{11}\)

The cost of precaution, to be borne by the injurers, can either be \(c_0(x)\) or \(c_1(x)\), where \(x\) is the level of precautions.\(^{12}\) Precautions reduce the "cost of harm" for the victims, which can be either \(\ell_0(x)\) or \(\ell_1(x)\), with \(\ell'_0 < 0\), \(\ell'_1 < 0\). The cost of harm represents a quantification of the risk of harm for the victims: it includes expected harm, \(p_0(x) h\) or \(p_1(x) h\), and a risk premium, due to the random nature of harm. Figure 1 illustrates.

Once uncertainty has unfolded and the costs and benefits of precaution are known, a precautionary standard will be enforced. If cost benefit analysis is applied, the conventional standards \(x^\circ_0\) and \(x^\circ_1\) will be selected, balancing marginal benefits and marginal costs. Depending on which state of the world arises, precaution costs will either be \(c_0(x^\circ_0)\) or \(c_1(x^\circ_1)\), while the cost of harm for the victims will either be \(\ell_0(x^\circ_0)\) or \(\ell_1(x^\circ_1)\).

\(^{11}\)In Shavell (2014b), the uncertainty is about the harm side only: the activity can either be harmful or not. The emphasis is on the attenuation of the ex-ante risk borne by the injurers.

\(^{12}\)In this paper, I do not focus on the costs of change, relating to the need to "retrofit" long lasting prevention technologies. When I say that a technology entails lower costs, I mean all costs, including the cost of adoption and adaptation. For a thorough analysis of the impact of retrofitting costs, see Shavell (2008). Dari-Mattiacci and Franzoni (2014) shows how negligence law should be designed to favor the adoption of new prevention technologies.
Figure 1: The attenuation principle

From an ex-ante perspective, this situation entails uncertainty for both the injurers and the victims. The size of ex-ante risk they face, as I will show in the paper, is measured by the wedge in the prospective payoffs: $|c_0(x_0^c) - c_1(x_1^c)|$ for the injurers, and $|\ell_0(x_0^c) - \ell_1(x_1^c)|$ for the victims. To fix ideas, I assume that $c_0(x_0^c) > c_1(x_1^c)$ and $\ell_0(x_0^c) > \ell_1(x_1^c)$.

As mentioned in the introduction, the government can mitigate the ex-ante uncertainty by means of different tools, including subsidies, exemptions, grandfathering and so on. Here, the focus is on the policy itself: how should the precautionary standards be fixed, to mitigate the ex-ante uncertainty borne by the parties?

Let us consider the impact of a variation in the conventional standards on social welfare. By reducing the standard $x_0^c$, the policy maker invites the injurers to forgo precautionary measures that are more beneficial than costly. However, the resulting
What is substantial, instead, is the impact on the size of the ex-ante risk borne by the parties. The uncertainty burden of the injurers decreases, since \( c(x_0^c) \) decreases, while the ex-ante uncertainty burden of the victims increases, since \( \ell_0(x_0) \) increases, and exactly by the same amount by which \( c(x_0) \) decreases. The net effect, however, is not null, since parties are likely to have a different capacity to bear additional risk. This is because they might have different attitudes towards risk and because the size of the risk they already bear is different (in general, the higher the risk already borne, the more costly the addition of risk). So, if injurers are less able to bear additional risk, the standard \( x_0 \) should decrease, and the other way around if victims are less able to bear additional risk.

On the basis of similar considerations, the standard \( x_1^c \) also should be be modified. If injurers are less able to bear additional risk, the standard \( x_1^c \) should increase, and the other way.

The basic lesson learned from these observations is that, in order to attenuate ex-ante uncertainty, conventional cost-benefit analysis should be modified. In general, the direction of change of a standard will depend on the following factors: i) the sign of the shocks affecting the parties (e.g., whether \( c_0(x_0^c) > c_1(x_1^c) \) and \( \ell_0(x_0^c) > \ell_1(x_1^c) \)); ii) the size of the shock (that is \( |c_0(x_0^c) - c_1(x_1^c)| \) and \( |\ell_0(x_0^c) - \ell_1(x_1^c)| \)), iii) the parties’ disposition towards risk. Note that what matters is not the aggregate size of the risk, but the individual size (i.e., the risk that each individual bears: as in regular risk sharing, the fact that harm is spread to many parties is an advantage).

The precise amount by which \( x_0^c \) and \( x_1^c \) should change depends on the degree of risk aversion of the parties and the size of the risk they bear. Additionally, if the marginal cost and marginal harm curves are highly inelastic (marginal costs and marginal harm are nearly constant), the cost of distorting the conventional policy is smaller, and the optimal distortion is larger.\(^{14}\)

The analysis can be easily extended to a multiperiod framework and to non-binary

\(^{13}\)This is an application of the Envolepe Theorem.

\(^{14}\)The focus on state-contingent policy differentiates the current contribution from the classic problem of policy making under uncertainty thoroughly explored in environmental policy (started by the pioneering contribution of Weitzman (1974)). In the latter tradition, uncertainty is a concern because it leads to costly mistakes. In my model, uncertainty is a concern because it creates costly risk.
states of the world. The larger the shock suffered by the parties, and the stronger the attenuation requested.

The cursory analysis of the previous paragraphs is based on a remarkable feature: the victims’ ex-ante uncertainty enters social welfare in a way that is fully symmetrical to that of the injurers. The reason why this is remarkable has to do with the nature of the uncertainty they bear. For them, the uncertainty is about the probability of a loss, not an actual loss. Under Expected Utility theory, variability in future outlays causes a loss of utility, while variability in probabilities does not. So, why should we care about the ex-ante uncertainty borne by the victims?

In the ex-post stage, when the probability of harm becomes known, the optimal policy will balance costs and benefits of precaution. Since victims are subject to a stochastic prospect of harm, they bear costly risk. The ex-post efficient level of precaution will take this risk into account: at the optimal standards, one dollar of precaution should reduce expected harm plus the risk premium of the victims by one dollar. The conventional policy offers some ex-post insurance to the victims. Consider now a modification of the conventional standards that induces a mean preserving contraction of the probabilities of harm. Such a change does not affect the utility of the victims. This means that the total risk they are subject to is unaffected. Since the ex-ante risk decreases, the ex-post risk must increase (I formally prove this statement in Appendix A1). This implies that fewer resources are devoted to the mitigation of ex-post risk: victims are offered ex-ante insurance in exchange for a reduction in (costly) ex-post insurance. Leaving aside the possible adverse effect on the risk borne by the injurers, the mean preserving contraction entails an increase in social welfare.

The following sections provide a formal analysis of the attenuation principle.

3 Regulation of behavior

Let us consider the case in which both the costs and the probability of harm associated with a certain activity are uncertain. They will only be known at a future date. At that date, when uncertainty has unraveled, a suitable standard of behavior will be applied.
The standard will be enforced either through pecuniary and criminal sanctions, or simple negligence law.

With probability \( q \) the cost of taking precautions is \( c_0(x) \) and the probability that a victim suffers harm is \( p_0(x) \), with \( c_0'(x) > 0 \), \( c_0'(0) = 0 \), \( \lim_{x \to \infty} c_0'(x) = \infty \), \( c_0''(x) \geq 0 \), \( p_0'(x) < 0 \), and \( p_0''(x) \geq 0 \).

With probability \( 1 - q \), the cost of taking precautions will be \( c_1(x) \) and the probability that a victim suffers harm is \( p_1(x) \), where \( c_1(x) \) and \( p_1(x) \) meet the regularity conditions listed above.

The magnitude of harm is fixed and equal to \( h \) for each victim.\(^{15}\) There are \( n_V \) symmetric victims and \( n_I \) symmetric injurers. The policy maker sets the standards \( x_0 \) and \( x_1 \) before uncertainty has been resolved and commits not to change them.

In line with this paper’s introductory observations, I assume that insurance is not available. So, injurers are subject to the risk associated with the variability of precaution costs, while victims are subject to the risk of harm, with a probability that is itself uncertain.

We can now turn to the optimal policy. As noted in the introduction, the policy maker will find it optimal to deviate from the prescriptions of conventional cost-benefit analysis. In order to see in which direction policy changes, I first analyze the "conventional" standards, that is the standards that are optimal ex-post, when uncertainty has unraveled.\(^{16}\)

Let us suppose that state 0 has occurred. Let us consider the payoff of the victims. Given the level of precaution taken by the injurers, the cost of harm for each victim is

\[
\ell_0(x_0) = p_0(x_0) h + R P^V_0(x_0),
\]

where \( R P^V_0(x_0) \geq 0 \) is the risk premium that victims associate to the prospect of losing \( h \) with probability \( p_0(x_0) \). \( \ell_0(x_0) \) is the amount that victims would be willing to pay to

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\(^{15}\)Results can be easily extended to the case in which also victims invest in precautions (bilateral accidents) and precautions affect the magnitude of harm (instead of its probability).

\(^{16}\)Shavell (2014b) uses the same definition. However, since in his model victims can buy insurance once uncertainty has unraveled, his conventional standards coincide with the standards that would emerge if all parties were risk neutral.
get full insurance against harm. This amount depends on the probability of harm, the magnitude of harm, and the disposition towards risk of the victims. For our purposes, we do not need to explicitly calculate it. One can easily see that if the standard of care \( x_0 \) increases, the probability of harm decreases and \( \ell_0 (x_0) \) decreases. If \( h \) increases, \( \ell_0 (x_0) \) increases.

The optimal standard for the conventional case can be found by minimizing social loss after uncertainty has unraveled:

\[
L_0 (x_0) = n_I c_0 (x_0) + n_V \ell_0 (x_0).
\]

The optimal conventional standard \( x_0^c \) must satisfy \( L_0' (x_0^c) = 0 \) and thus:

\[
n_I c_0' (x_0^c) = -n_V \ell_0' (x_0^c) = -n_V \left[ p_0' (x_0^c) h + RP_0^{V'} (x_0^c) \right]. \tag{1}
\]

Eq. (1) is a classic application of cost-benefit analysis to risk prevention: an additional dollar spent in prevention should reduce the cost of harm (expected harm + risk premium) by one dollar. In general, we do not know if risk aversion calls for a higher or lower standard (\( RP_0^{V'} (x) \) can be positive or negative).

Note that the conventional standard \( x_0^c \) increases if \( n_V \) increases or \( n_I \) decreases: social marginal costs increase if the number of injurers is larger, social marginal benefits increase if the number of potential victims is larger.

Similarly, the optimal conventional standard \( x_1^c \) must satisfy \( L_1' (x_1^c) = 0 \) and thus:

\[
n_I c_1' (x_1^c) = -n_V \ell_1' (x_1^c) = -n_V \left[ p_1' (x_1^c) h + RP_1^{V'} (x_1^c) \right]. \tag{2}
\]

Note that \( x_1^c \) might be larger or smaller than \( x_0^c \).

Let us consider now the optimal standards from an ex-ante perspective. Again, the

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17 The analysis of this paper is greatly simplified by the use of certainty equivalents (instead of expected utility functions). In Appendix A1, I show that the two methods are equivalent.

18 Note that as the probability of harm decreases, \( RP_0^{V'} (x) \) can decrease or increase. \( \ell_0 (x) \), instead, unambiguously decreases (at an increasing rate).

19 \( RP_0^{V'} (x) \) is negative, however, if \( p_0 (x) \) is sufficiently small (less than 1/2 under quadratic utility). See Jullien et al. (1999).
optimal policy is obtained by minimizing the sum of the ex-ante losses. For the injurer, the ex-ante loss (i.e., the amount she would be willing to spend to get full insurance against precaution costs) is

$$\ell_{I}^{ex-ante} (x_0, x_1) = q c_0 (x_0) + (1 - q) c_1 (x_1) + R_I,$$

where $R_I$ is the risk premium due to ex-ante uncertainty about precaution costs. $R_I$ goes up if the wedge between $c_0 (x_0)$ and $c_1 (x_1)$ increases, and if the injurer is more averse to risk.\footnote{Note that also firms tend to display risk aversion (and purchase insurance, when available). Several explanations for this fact have been provided, including the cost of bankruptcy (which leads firms to avoid very adverse outcomes), the cost of external funds (which might be needed in adverse states), asymmetric information (the firm’s behavior reflects the risk aversion of directors and managers whose remuneration depends on the firm’s performance), a convex tax schedule, and debt overhang (the underinvestment caused by debt can be reduced if risks are managed).}

For the victims, a similar equation applies (see Appendix A1):

$$\ell_{V}^{ex-ante} (x_0, x_1) = q \ell_0 (x_0) + (1 - q) \ell_1 (x_1) + R_V. \quad (3)$$

Ex-ante loss includes the mean cost of harm ($\ell_0 (x_0)$ and $\ell_1 (x_1)$) and the risk premium attendant with ex-ante uncertainty. Note that $R_V$ decreases if the wedge between $\ell_1$ and $\ell_0$ decreases, and if victims are more averse to risk (given $\ell_1$ and $\ell_0$, $R_V$ increases if $v$ is subject to a concave monotone transformation).

We are now ready to calculate the ex-ante optimal policy. The optimal standards should minimize ex-ante social loss:

$$L^{ex-ante} (x_0, x_1) = n_I \ell_{I}^{ex-ante} (x_0, x_1) + n_V \ell_{V}^{ex-ante} (x_0, x_1)$$

$$= q L_0 (x_0) + (1 - q) L_1 (x_1) + n_I R_I + n_V R_V. \quad (4)$$

Ex-ante social loss includes the mean value of social loss over the different states, and the risk premiums for injurers and victims due to ex-ante uncertainty.

The assumptions about the cost functions and the probabilities of harm guarantee that the optimal standards lie in the interior. I further assume that $L^{ex-ante} (x_0, x_1)$ is strictly quasi-concave.
Let us see how an increase in \(x_0\) affects \(L^{ex-ante}(x_0,x_1)\). We have
\[
\frac{\partial L^{ex-ante}(x_0,x_1)}{\partial x_0} = qL'_0(x_0) + n_I \frac{\partial R_I}{\partial c_0}(x_0)c'_0(x_0) + n_V \frac{\partial R_V}{\partial \ell_0}(x_0)\ell'_0(x_0).
\]

An increase in the standard \(x_0\) affects ex-post social welfare \(L_0(x_0)\), and the ex-ante uncertainty burden of the parties.

To determine the direction in which the standards should be modified, let us consider the sign of marginal ex-ante social loss at the conventional levels. At \(x_0 = x_0^c\), \(x_1 = x_1^c\), we have \(L'_0(x_0) = 0\) and \(n_I\ c'_0(x_0^c) = -n_V\ \ell'_0(x_0^c)\), so:
\[
\frac{\partial L^{ex-ante}(x_0^c,x_1^c)}{\partial x_0} > 0 \iff \frac{\partial R_I}{\partial c_0}(x_0^c) > \frac{\partial R_V}{\partial \ell_0}(x_0^c).
\]

Whether the conventional standard \(x_0^c\) should decrease or increase depends on how parties’ risk premiums change in response to a change in the ex-post payoff.

Similarly, we get
\[
\frac{\partial L^{ex-ante}(x_0^c,x_1^c)}{\partial x_1} > 0 \iff \frac{\partial R_I}{\partial c_1}(x_1^c) > \frac{\partial R_V}{\partial \ell_1}(x_1^c).
\]

We have thus proved the following.

**Proposition 1** *Optimal standards under uncertainty.*

a) The standard \(x_0^c\) should be decreased if, and only if, \(\frac{\partial R_I}{\partial c_0}(x_0^c) > \frac{\partial R_V}{\partial \ell_0}(x_0^c)\).

b) The standard \(x_1^c\) should be decreased if, and only if, \(\frac{\partial R_I}{\partial c_1}(x_1^c) > \frac{\partial R_V}{\partial \ell_1}(x_1^c)\).

The optimal attenuation policy requires that the conventional standards be modified to mitigate the individual ex-ante uncertainty borne by injurers and victims.

We have
\[
\frac{\partial R_I}{\partial c_0}(x_0^c) > 0 \iff c_0(x_0^c) > c_1(x_1^c), \quad \frac{\partial R_V}{\partial \ell_0}(x_0^c) > 0 \iff \ell_0(x_0^c) > \ell_1(x_1^c)
\]
\[
\frac{\partial R_I}{\partial c_1}(x_1^c) > 0 \iff c_0(x_0^c) < c_1(x_1^c), \quad \frac{\partial R_V}{\partial \ell_1}(x_1^c) > 0 \iff \ell_0(x_0^c) < \ell_1(x_1^c).
\]
So, if \( c_0 (x_0^c) > c_1 (x_1^c) \) and \( \ell_0 (x_0^c) < \ell_1 (x_1^c) \), a slight reduction of \( x_0 \) and a slight increase in \( x_1 \) decrease the ex-ante risk of both injurers and victims. Similarly, if \( c_0 (x_0^c) < c_1 (x_1^c) \) and \( \ell_0 (x_0^c) > \ell_1 (x_1^c) \), a slight increase in \( x_0 \) and a slight reduction of \( x_1 \) are clearly welfare improving.

The direction of change cannot be unambiguously determined when injurers and victims are subject to shocks of the same sign. In such situations, the direction of change will depend on which party is in a better position to bear additional (ex-ante) risk. This will in general depend on their disposition towards risk and the magnitude of the risk they are already bearing. Note that for this determination, the number of injurers and victims does not matter.

To get some further insight, let us assume that the ex-ante risk is "small." This allows us to use a Arrow-Pratt approximation for the ex-ante risk premiums:

\[
R_V = \frac{1}{2} \alpha_V q (1 - q) \left[ \ell_0 (x_0) - \ell_1 (x_1) \right]^2, \\
R_I = \frac{1}{2} \alpha_I q (1 - q) \left[ c_0 (x_1) - c_1 (x_1) \right]^2,
\]

where \( \alpha_V \) and \( \alpha_I \) are the indexes of Absolute Risk Aversion of victims and injurers, respectively, and \( q (1 - q) \left[ \ell_0 (x_0) - \ell_1 (x_1) \right]^2 \) and \( q (1 - q) \left[ c_0 (x_1) - c_1 (x_1) \right]^2 \) the (ex-ante) variances of their losses.\(^{21}\)

The main results takes this simpler shape.

*Optimal standards (small risk).*

a) If \( \alpha_I \left[ c_0 (x_0^c) - c_1 (x_1^c) \right] > \alpha_V \left[ \ell_0 (x_0^c) - \ell_1 (x_1^c) \right] \), ex-ante uncertainty conveys a decrease in the standard \( x_0^c \) and an increase in the standard \( x_1^c \);

b) If \( \alpha_I \left[ c_0 (x_0^c) - c_1 (x_1^c) \right] < \alpha_V \left[ \ell_0 (x_0^c) - \ell_1 (x_1^c) \right] \), ex-ante uncertainty conveys an increase in the standard \( x_0^c \) and a decrease in the standard \( x_1^c \).

The use of Arrow-Pratt approximation allows for a simple determination of the direction of change, that does not depend on the income levels of the parties.\(^{22}\) When

\(^{21}\)From auto collision insurance choices of US households, Barseghyan et al. (2013) get a baseline estimate of \( \alpha \in [0.002, 0.008] \).

\(^{22}\)In general, disposition towards risk depends on income. This does not imply that Proposition 1 is indeterminate. In fact, this Proposition provides a "local" result: it tells us in which direction standards
the shocks affecting the parties have the same sign, the direction in which the standards should be modified depends on two factors: i) the disposition of injurers and victims towards risk, captured by the indexes $\alpha_I$ and $\alpha_V$, ii) the size of the shocks $|c_0 (x_0^e) - c_1 (x_1^e)|$ vs. $|\ell_0 (x_0) - \ell_1 (x_1)|$.

### 4 Liability for harm

Let us now consider the case in which injurers are liable for the harm caused. The policy maker decides the level of damages to be paid to each victim ($d_0$ if state 0 occurs, $d_1$ if state 1 occurs). Each injurer faces $n = n_V/n_I$ victims. Damages are independent of the level of care taken (i.e., strict liability applies), and liability is clearly assigned to the responsible injurer. Damages can undercompensate (e.g., pain and suffering are not included, or caps are imposed) or overcompensate (they are generously calculated or they include punitive elements). Each victim suffers harm $h$ with probability either $p_0 (x)$ or $p_1 (x)$.

Let us start again with the conventional levels of damages, i.e. the damages that would be optimal ex-post, when uncertainty about the precaution costs and the probability of harm has unfolded.

Let us focus on state 0. The cost of harm for each injurer is

$$\ell_I^0 (x_0) = c_0 (x_0) + p_0 (x_0) nd_0 + RP_I^0 (nd_0),$$

where $RP_I^0 (nd_0)$ is the risk premium attendant with the risk of bearing liability $nd_0$ with probability $p_0 (x_0)$.

The level of care $x_0^I$ is decided by the injurer to minimize her cost of harm:

$$x_0^I = \arg \min_{x_0} \ell_I^0 (x_0),$$

should be modified given the income levels of the parties. It characterizes a "Pareto improvement" rather than a "Pareto optimum."

---

23Accidents are assumed to be perfectly correlated. Optimal liability law with imperfectly correlated accidents is studied in Franzoni (2016). Correlation impacts the size of $RP_I^0 (nd_0)$.
and thus

\[ c'_0(x'_0) = - \left[ p'_0(x'_0) n d_0 + \frac{\partial R P'_0(n d_0)}{\partial x_0} \right]. \]

One dollar spent by the injurer in precaution reduces liability and the attendant risk premium by one dollar.

The costs of harm for each victim is:

\[ \ell'_0(x'_0) = p_0(x'_0)(h - d_0) + R P'_0(h - d_0), \]

where \( R P'_0(h - d_0) \) is the risk premium attendant with the prospect of bearing uncompensated harm \( h - d_0 \) with probability \( p_0(x'_0) \). The cost of harm increases with \( h - d_0 \) and \( p_0(x'_0) \) (see Appendix A3).

The optimal conventional policy can be found by minimizing social loss (per injurer):

\[ L_{SL}^c(d_0) = \ell'_0(x'_0) + n \ell'_0(x'_0). \]

Thus, conventional damages \( d^c_0 \) should solve:

\[ \frac{L_{SL}^c(d^c_0)}{\partial d_0} = \frac{\partial \ell'_0(x'_0)}{\partial d_0} + n \frac{\partial \ell'_0(x'_0)}{\partial d_0} = 0. \]  

(7)

Note that damages affect both the allocation of risk between the parties and the level of precaution taken by the injurer.

Optimal (conventional) damages satisfy: \( 0 < d^c_0 \leq h \) (see Appendix A3). That optimal damages are under-compensatory \( (d^c_0 < h) \) when victims are risk averse is a classic result in liability theory (see Shavell (1982)). The intuition is the following: when damages are compensatory \( (d^c_0 = h) \), the injurer fully internalizes the negative impact she exerts on the victims, so incentives to take precautions are properly set.

The allocation of risk, however, is not optimal, because all the risk is borne by the injurer. The allocation of risk can be improved at the margin, with a negligible effect on incentives, by reducing the level of damages and by shifting a small amount of the loss on the victims. The impact on the risk burden of the victims is negligible (for small losses, people behave as if they were risk neutral), while the impact on the risk burden
of the injurer is substantial.  

Similar definitions and results apply to state 1.

Let us consider the optimal policy before uncertainty unravels. Ex-ante social loss can be written, as in Section 3, as

\[ L^{\text{ex-ante}}(d_0, d_1) = qL_{0}^{SL}(d_0) + (1 - q)L_{1}^{SL}(d_1) + R_I + nR_V, \]  

(8)

where \( R_I \) and \( R_V \) are the risk premiums of injurer and victims, respectively, due to ex-ante uncertainty.

If we consider marginal social welfare at the conventional damages \( d_{c0} \) and \( d_{c1} \), we get:

\[
\frac{\partial L^{\text{ex-ante}}(d_{c0}, d_{c1})}{\partial d_0} = \frac{\partial R_I}{\partial l^I_0(x^I_0)} \frac{\partial l^I_0(x^I_0)}{\partial d_0} + n \frac{\partial R_I}{\partial l^I_0(x^I_0)} \frac{\partial l^I_0(x^I_0)}{\partial d_0},
\]

\[
\frac{\partial L^{\text{ex-ante}}(d_{c0}, d_{c1})}{\partial d_1} = \frac{\partial R_I}{\partial l^I_1(x^I_1)} \frac{\partial l^I_1(x^I_1)}{\partial d_1} + n \frac{\partial R_I}{\partial l^I_1(x^I_1)} \frac{\partial l^I_1(x^I_1)}{\partial d_1}.
\]

So, using the first order conditions (7), we get the following.

**Proposition 2 Liability for harm caused.**

a) Damages \( d_{0}^c \) should be decreased if, and only if, \( \frac{\partial R_I}{\partial l^I_0(x^I_0)} > \frac{\partial R_V}{\partial l^I_0(x^I_0)} \);

b) Damages \( d_{1}^c \) should be decreased if, and only if, \( \frac{\partial R_I}{\partial l^I_0(x^I_0)} > \frac{\partial R_V}{\partial l^I_0(x^I_0)} \).

Proposition 2 mimics Proposition 1. Optimal attenuation requires a modification of the ex-post optimal policy. Damages should be decreased if injurers are subject to a negative shock while victims are subject to a positive shock (and the other way around).

When parties are subject to shocks with the same sign, then attenuation should take care of the party that benefits most from a reduction in ex-ante risk.

Using an Arrow-Pratt approximation (for ex-ante uncertainty), we get:

Optimal damages (small risk).

\(^{24}\)This result breaks down if victims display first-order risk aversion (see Franzoni (2017)).
a) If \( \alpha_I [\ell_I^0 (x_I^0) - \ell_I^1 (x_I^1)] > \alpha_V [\ell_V^0 (x_V^0) - \ell_V^1 (x_V^1)] \), ex-ante uncertainty conveys a decrease in damages \( d_0^I \) and an increase in damages \( d_1^I \);

b) If \( \alpha_I [\ell_I^0 (x_I^0) - \ell_I^1 (x_I^1)] < \alpha_V [\ell_V^0 (x_V^0) - \ell_V^1 (x_V^1)] \), ex-ante uncertainty conveys an increase in damages \( d_0^I \) and a decrease in damages \( d_1^I \).

Under reasonable conditions, we should expect injurers to bear greater ex-ante uncertainty than the victims. For the injurers, ex-ante uncertainty stems from the variability of precaution costs, liability expenses, and attendant risk premium. For the victims, ex-ante uncertainty only includes the variability of uncompensated harm and attendant risk premium. So, optimal attenuation is likely to be geared towards the injurers.

In Appendix 3, I show that optimal damages may be overcompensatory: \( d_0^* > h \) (or \( d_1^* > h \)). Under risk neutrality, optimal damages are perfectly compensatory. Under risk aversion, a deviation from the risk-neutral benchmark is desirable, for a better allocation of risk. In the neighborhood of \( d_0^* = d_1^* = h \), a small change in damages has no effect on the victims, who are fully insured against harm. So, the relevant impact is on the risk burden of the injurer. If ex-post risk is what matters most, then damages should be reduced (both \( d_0^* \) and \( d_1^* \)). If ex-ante risk is what matters most - because ex-ante uncertainty is substantial - then damages should be overcompensatory in the presence of a positive shock, and under-compensatory in the presence of a negative shock.

5 Corrective taxation

Let us now suppose that the government relies on corrective taxation to encourage precaution (or, equivalently, a reduction in emissions). To simplify the exposition, I focus on simple linear taxes:

\[
T_0 (x_0) = t_0 (X - x_0), \text{ and } T_1 (x_1) = t_1 (X - x_1).
\]
$X$ captures the "baseline" level of precaution. If the injurer does less than $X$, she pays a tax; if she does more, she gets a subsidy. In the case of an emission tax, $X$ represents the fixed emissions allowance (possibly grandfathered). So, this mechanism also captures a cap-and-trade policy.\footnote{Shavell (2014b) considers the case in which the tax depends on the probability of harm, $T_0 (x) = t_0 p_0 (x)$, and victims are risk-neutral. The optimal conventional level of $t_0$ is $h$, so that $T_0^* (x) = p_0 (x) h$. This formulation mimics the liability for harm case (if injurers can buy liability insurance). Non-linear taxes are theoretically superior (see Kaplow and Shavell (2002)), but they entail large administrative costs. The current policy debate focuses on linear taxes and cap-and-trade (possibly with a collar).}

$X$ is assumed to be fixed. If $X$ could vary across states, it could be used to provide perfect insurance to the injurers (with a large $X$ in bad states, and a small $X$ in good states). If that were the case, the only remaining concern for the policy maker would be to provide (partial) insurance to the victims. This case can be captured in the present model by setting $\alpha_I = 0$.

Let us focus on state 0. The ex-post cost of harm for the injurer is:

$$
\ell^I_0 (x^I_0) = c_0 (x^I_0) + t_0 (X - x_0) .
$$

So the injurer selects a level of precaution $x^I_0$ such that

$$
\ell^I_0 (x^I_0) = t_0 .
$$

(9)

We have

$$
\frac{\partial \ell^I_0 (x^I_0)}{\partial t_0} = \frac{\partial \ell^I_0 (x^I_0)}{\partial x^I_0} \frac{\partial x^I_0}{\partial t_0} + \frac{\partial \ell^I_0 (x^I_0)}{\partial t_0} \bigg|_{x_0=x^I_0} = X - x_0 ,
$$

(10)

since $\frac{\partial \ell^I_0 (x^I_0)}{\partial x^I_0} = 0$. As the tax rate increases, the cost of harm for the injurer can increase (if $X > x_0$) or decrease (if $X < x_0$). In the case of the emission fee, for instance, it depends on the whether the injurer emits more or less than the fixed allowance.

The costs of harm for each victim is:

$$
\ell^V_0 (x^I_0) = p_0 (x^I_0) h + RP^V_0 (x^I_0) ,
$$
where $R_P^V (x_0^i)$ is again the risk premium attendant with the harm prospect, with

\[
\frac{\partial \ell_0^V (x_0^i)}{\partial t_0} = \frac{\partial x_0^i}{\partial t_0} \left[ p_0' (x_0^i) h + R_P^V (x_0^i) \right] < 0. \tag{11}
\]

Ex-post social loss includes the loss to the injurer, the loss to the victims, minus the tax (that can be redistributed to society at large):\(^{26}\)

\[
L_0 (t_0) = c_0^i (x_0^i) + n_p^V (x_0^i) - T_0 (x_0) = c_0 (x_0^i) + n [ p_0 (x_0^i) h + R_P^V (x_0^i) ] .
\]

Note that the tax only affects social loss indirectly, by means of its impact on $x_0^i$.

The optimal conventional tax solves:

\[
\frac{\partial L_0 (t_0)}{\partial t_0} = \frac{\partial x_0^i}{\partial t_0} \left[ c_0' (x_0^i) + n p_0' (x_0^i) h + n R_P^V (x_0^i) \right] = 0. \tag{12}
\]

At the conventional optimum, marginal costs have to be equal to marginal benefits of precaution. The latter include the impact of precautions on the uncertainty burden of the victims (the latter may be positive or negative, as remarked in Section 3). So, using (9): $t_0^c = c_0^i (x_0^i) = - n p_0^i (x_0^i) h - n R_P^V (x_0^i)$. If the number of victims increases, the marginal benefit of precaution increases and $t_0^c$ increases.

In state 1, by similarity, we have: $t_1^c = c_1^i (x_1^i) = - n p_1^i (x_1^i) h - n R_P^V (x_1^i)$.

Let us move to the ex-ante stage. Ex-ante social loss can be written as:

\[
L^{ex-ante} (t_0, t_1) = q L_0 (t_0) + (1 - q) L_1 (t_1) + R_I + n R_V,
\]

where $R_I$ and $R_V$ are the ex-ante risk premiums of injurer and victims, respectively.

\(^{26}\)If the tax were redistributed to the victims only, then the corrective taxation regime would resemble the liability regime, bar for the fact that the tax is levied on all injurers and not only on those who happen to cause an accident. So, the tax would provide accident insurance to the injurers. The tax could also be redistributed only to the injurers. Such an outcome can be mimicked by suitably changing the benchmark $X$. 

21
Let us consider the impact of a change in $t_0$, given $t_0^c$ and $t_1^c$. We have

$$\frac{\partial L^{ex-ante}(t_0^c, t_1^c)}{\partial t_0} = q \frac{\partial L_0(t_0)}{\partial t_0} + \frac{\partial R_I}{\partial x_I(x_0^I)} \frac{\partial x_I^I}{\partial t_0} + n \frac{\partial R_V}{\partial x_V(x_0^V)} \frac{\partial x_V^V}{\partial t_0}.$$

Thus, from (10), (11) and (12):

$$\frac{\partial L^{ex-ante}(t_0^c, t_1^c)}{\partial t_1} > 0 \iff \frac{\partial R_I}{\partial x_I(x_0^I)} (X - x_0^I) > \frac{\partial R_V}{\partial x_V(x_0^V)} \frac{\partial x_V^V}{\partial t_0} \left[ -np_0'(x_0^I) \frac{\partial x_0^I}{\partial t_0} h - nR_0^{V'}(x_0^I) \right]$$

$$\iff \frac{\partial R_I}{\partial x_I(x_0^I)} (X - x_0^I) > \frac{\partial R_V}{\partial x_V(x_0^V)} \frac{\partial x_V^V}{\partial t_0} x_0^c.$$  (13)

Figure 2 illustrates. In contrast to the cases analyzed in the previous sections, here a policy change impacts the payoffs of the parties in a non-symmetric way.

![Figure 2. Corrective taxation](image)

A decrease in the tax increases the (ex-post) payoff the injurer by the amount $X - x_0^c$ - the horizontal shaded area. At the same time, a decrease in the tax increases the cost
of harm for the victims by an amount that depends on the sensitivity of precautions to the tax - the vertical shaded area. If marginal precaution costs are very steep, a reduction in the tax has a small effect on the victims.

Inequality (13) can be rewritten as
\[
\frac{\partial L^{\text{ex-ante}} (t^c_0, t^c_1)}{\partial t_1} > 0 \iff \frac{\partial R_I}{\partial t_0} \frac{X - x^I_0}{x^I_0} > \frac{\partial R_V}{\partial t_0} \varepsilon_t^{x_0}_0,
\]
where
\[
\varepsilon_t^{x_0}_0 = \frac{\partial x^I_0}{\partial t_0} \frac{t^c_0}{x^I_0} = \frac{1}{c''_0 (x^I_0)} \frac{t^c_0}{x^I_0} > 0
\]
is the elasticity of precautions with respect to the tax rate (from 9).

Similarly, we get
\[
\frac{\partial L^{\text{ex-ante}} (t^c_0, t^c_1)}{\partial t_1} > 0 \iff \frac{\partial R_I}{\partial t_1} \frac{X - x^I_1}{x^I_1} > \frac{\partial R_V}{\partial t_1} \varepsilon_t^{x_1}_1.
\]
We have thus proved the following.

**Proposition 3 Corrective taxation.**

a) The tax \(t^c_0\) should be decreased if, and only if, \(\frac{\partial R_I}{\partial t_0} \frac{X - x^I_0}{x^I_0} > \frac{\partial R_V}{\partial t_0} \varepsilon_t^{x_0}_0\).

b) The tax \(t^c_1\) should be decreased if, and only if, \(\frac{\partial R_I}{\partial t_1} \frac{X - x^I_1}{x^I_1} > \frac{\partial R_V}{\partial t_1} \varepsilon_t^{x_1}_1\).

The inequalities of Proposition 3 are substantially different from those obtained in the previous sections. The direction of change of the conventional policy depends, in addition to the sign of the shock (positive or negative), on the impact of an increase in the tax rate on the outlays of the injurer and on the elasticity of precaution.\(^{27}\) If precautions are relatively inelastic (marginal costs are steep), policy should only focus on mitigating the risk borne by the injurers.

If the injurer is risk averse and the victims are risk neutral, the tax outlays should be decreased under a negative shock (for the injurer), while they should be increased.

\(^{27}\)Under a cap-and-trade system, a change in the tax is equivalent to a change in the price of the emission permits.
under a positive shock. The decrease in the tax outlays is obtained by a decrease in $t_0$ if $X > x_0^I$ (tax) and the other way around if $X < x_0^I$ (subsidy).\textsuperscript{28}

If the injurer is risk neutral and victims are risk averse, the tax rate should be increased under a negative shock (for the victims), while it should be decreased under a positive shock. This case also captures the situation in which the baseline level of precaution $X$ can be adjusted according to the circumstances.

Using an Arrow Pratt approximation, we get the following.

**Corrective taxation (small risk)**

1. If $\alpha_I \left[ \ell^I_0 \left( x_0^I \right) - \ell^I_1 \left( x_1^I \right) \right] \frac{x - x_0^I}{x_0^I} > \alpha_V \left[ \ell^V_0 \left( x_0^V \right) - \ell^V_1 \left( x_1^V \right) \right] \varepsilon^0_I$, ex-ante uncertainty conveys a decrease in the tax $t_0$. In the opposite case, $t_0$ should increase.

2. If $\alpha_I \left[ \ell^I_0 \left( x_0^I \right) - \ell^I_1 \left( x_1^I \right) \right] \frac{x - x_1^I}{x_1^I} < \alpha_V \left[ \ell^V_0 \left( x_0^V \right) - \ell^V_1 \left( x_1^V \right) \right] \varepsilon^1_I$, ex-ante uncertainty conveys a decrease in the tax $t_1$. In the opposite case, $t_1$ should increase.

### 6 Extensions

The foregoing analysis focusses on the optimal attenuation given the policy instrument. It does not compare instruments themselves. Note that all instruments are equally efficient when injurers and victims are risk neutral. They differ substantially, instead, when parties are risk averse.

Under regulatory standards, injurers face uncertainty about precaution costs, while victims bear the risk of harm. Under liability, injurers face uncertainty about precaution costs and liability exposure, while victims bear the risk of being undercompensated. Under taxes, injurers face uncertainty about precaution costs and tax payments (if $X$ is fixed). Victims bear the risk of harm. These three allocations cannot be easily compared, bar for some polar cases.

**If injurers are risk neutral and victims risk averse, strict liability with full compensation yields the first best. Injurers bear all the risk, while victims are perfectly insured.**

\textsuperscript{28} For the tax case, this implies that "cost containment" policies (safety valves, collars, etc.) are efficient, even if they create ex-post distortions.
If victims are risk neutral and injurers are risk averse, regulation and taxation are likely to fare better. Under corrective taxation, if the allowance $X$ is suitably adjusted, injurers can be fully insured. So, corrective taxation dominates all other instruments.

The previous results show that the optimal policy should reduce the ex-ante uncertainty for the parties, even when this uncertainty is about the probability of harm. Thus, parties should be treated as if they were ambiguity averse, even when they are not. But, should they be?

In the model, the probabilities of legal change ($q$ and $1-q$) are taken as given. What is in these probabilities? If they were "objective" estimates, on which all observers agree, then ambiguity neutrality would make sense. Ambiguity aversion would have some bite, instead, in the case in which these probabilities were just subjective beliefs, about which parties do not have full confidence. In such a case, parties could legitimately cast rational doubt on them and adopt a "cautionary" approach, assigning a greater weight to the beliefs yielding the most adverse outcomes.

In Appendix A4, I consider the case in which parties are averse to a mean preserving spread of their beliefs. Ambiguity aversion makes the ex-ante uncertainty borne by the parties more costly. So, it calls for a greater attenuation effort. Note that under regulation and corrective taxation, only victims are subject to ambiguity, while under liability, both sides are.

In the model, parties have no access to private insurance. Would results change if private insurance were available? The answer tends to be negative. Insurance policies come with a loading factor, covering administrative costs, usually in the amount of 30-50% of the premium. Given a positive loading factor, optimal insurance contracts contain deductibles. Because of this, some risk remains on the parties. Thus, the results of the paper apply. If losses are correlated and insurance companies behave like

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29 In Appendix A4, I develop a simple sufficient condition for the dominance of regulation. This condition states that the payment of damages (in addition to precaution costs) increases the size of the ex-ante risk faced by the injurer.

30 See Gilboa and Marinacci (2013) for an introduction to the literature on ambiguity aversion. The impact of ambiguity aversion on liability law has been explored, among others, by Teitelbaum (2007) and Franzoni (2017).
risk-averse agents (e.g., because they face minimum capitalization requirements), the loading factor and the optimal deductibles are even higher.

Note, finally, that in the model, parties’ income is kept constant across states. In other words, variations in the parties’ payoffs are due only to changes in the "fundamentals" (precaution costs and probability of harm). It is clear that policy decision making could also provide individuals with (partial) insurance against income fluctuations. For instance, regulation could become softer when injurers are subject to a strong negative income shock while victims are not, and vice versa. Here, however, the question of why insurance is not provided through the tax system becomes more compelling.

7 Final remarks

The uncertainty of the timing and scope of new rules has long been a major cause for concern in the business community. More recently, this concern has been amplified by climate change. As the global temperature increases, additional mitigation policies are likely to be adopted that might cause the dismay of entire industries (and the joy of others). The scope and timing of these measures, however, remain extremely hard to predict due to the compound effects of political, natural, and scientific uncertainty (see IPCC (2014)). Literature has flourished, investigating pros and cons of alternative climate mitigation policies. One of the issues that is highly debated is whether industries and individuals should be compensated for the costs that mitigation policies inflict on them (see, for instance, Aldy et al. (2010)). At a fundamental level, this debate reverberates a long standing question: should parties be insured by the government?

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31This would represent a form of "countercyclical regulation" (see Masur and Posner (2017)). Fagan (2017) studies efficiency-enhancing (legal) stabilization rules.

32In a sense, attenuation provides a "local" solution to the insurance issue, as relief is provided by changing, at the margin, the level of welfare of the parties affected by the policy decision (injurers and victims). Taxes provide instead a "global" solution (risk is distributed over all members of society). The global solution tends to be theoretically preferable. It might fail, however, when the "local" problem is large enough. Correlated risks affecting a large section of the population cannot be perfectly spread.

33It is instructive to note, for example, that estimates of the social cost of a ton of carbon dioxide in 2050, consistent with a +2°C scenario, range from $40 to $470 (Stiglitz et al. (2017)). Policy uncertainty is commonly regarded as a major obstacle to the transition to a green economy.
against the losses caused by legal change? If so, how can such insurance best be provided?

This paper has investigated a specific tool that can be used to provide insurance to the parties affected by policy making, at little or no cost. This tool, first proposed by Steven Shavell, prescribes that policy decisions be distorted at the margin to attenuate the ex-ante risk borne by the parties. In the model developed in this paper, the attenuation policy is applied to regulation, liability law, and corrective taxation. In all three cases, attenuation increases social welfare.

The direction in which conventional policies should be changed depends on the sign of the shock parties are subject to (negative or positive), the size of the shock, and their disposition towards risk. These factors interact in a simple way. Their estimation requires little information. The corrective taxation case is somewhat more complicated because, here, the direction of change depends on the elasticity of precaution with respect to the tax rate, which might be somewhat harder to estimate.

The analysis of this paper focuses on the costs that ex-ante uncertainty places on the parties. It leaves many important issues aside. Among these, I should mention that it does not consider durable precautions (which are relevant if uncertainty unfolds gradually and the law is subject to repeated changes). By contrast, given its basic nature, it can be applied to a very large number of instances, essentially, to all policy decisions based on cost-benefit analysis.
Appendix

A1. Regulatory standards. The optimal policy is obtained from the maximization of the expected utility of the injurer given the expected utility of the victims (there are \( n_V/n_I = n \) victims per injurer), that is from

\[
\max_{x_0, x_1} EU_I (x_0, x_1) = qu (y_I - nt - c_0 (x_0)) + (1 - q) u (y_I - nt - c_1 (x_1))
\]

subject to

\[
EU_V (x_0, x_1) = q [(1 - p_0 (x_0)) v (y_V + t) + p_0 (x_0) v (y_V + t - h)] +
(1 - q) [(1 - p_1 (x_1)) v (y_V + t) + p_1 (x_1) v (y_V + t - h)]
\]

\[
= q \left[ \hat{EU}_0^V (x_0) \right] + (1 - q) \left[ \hat{EU}_1^V (x_1) \right] = r.
\]

\( y_I \) and \( y_V \) are the income levels of injurer and victims, respectively, \( r \) is the reference expected utility level of each victim, and \( t \) a money transfer from the injurer to each victim (which removes distributional concerns from the picture). The transfer takes place before uncertainty unravels.

Let us focus on the victims’ expected utility. Figure 2 illustrates (in the diagram the index \( V \) is omitted).

When the probability of harm is \( p_0 (x_0) \), the expected utility of the victim is \( \hat{EU}_0^V (x_0) \), and the certainty equivalent is \( y_V - \ell_0 (x_0) = y_V - p_0 (x_0) h - RP_0^V (x_0) \). The certainty equivalent includes the expected loss \( p_0 (x_0) h \) and the risk premium \( RP_0^V (x_0) \) due to the uncertainty about the occurrence of harm.

When the probability of harm is \( p_1 (x_1) \), the expected utility is \( \hat{EU}_1^V (x_1) \), and the certainty equivalent is \( y_V - \ell_1 (x_1) = y_V - p_1 (x_1) - RP_1^V (x_1) \).

From (15), it is clear that the ex-ante expected utility is equal to the expected utility when the probability of harm is \( \bar{\rho} = qp_0 (x_0) + (1 - q) p_1 (x_1) \). The certainty equivalent of \( EU_V (x_0, x_1) \) is \( y_V - z = y_V - \bar{\rho} h - RP_V (\bar{\pi}) \), where \( \bar{\pi} \) is the level of precaution that yields \( \bar{\rho} \).

The diagram shows that \( y_V - z \) can be obtained as the certainty equivalent of a lottery in which the income is \( y_V - \ell_0 (x_0) \) with probability \( q \), and \( y_V - \ell_1 (x_1) \) with probability \( 1 - q \). Specifically, \( y_V - z = y_V - q \ell_0 (x_0) - (1 - q) \ell_1 (x_1) - R_V \), where \( R_V \) is the risk premium due to the ex-ante uncertainty about the probability of harm. Note that \( R_V \) increases if \( \ell_0 \) decreases and if \( \ell_1 \) increases. Given \( \ell_1 \) and \( \ell_0 \), \( R_V \) increases if \( v \) is subject to a concave monotone
transformation.

So, we have

\[ y_V - ph - RP^V(x) \]
\[ = y_V - q \left( p_0(x_0) h - RP^V_0(x_0) \right) - (1 - q) \left( y_V - p_1(x_1) - RP^V_1(x_1) \right) - R_V, \]

that is

\[ RP^V(x) = qRP^V_0(x_0) + (1 - q)RP^V_1(x_1) + R_V. \] (17)

The total cost of uncertainty is equal to the mean cost of ex-post uncertainty plus the cost of ex-ante uncertainty.

In view of the previews observations, the constraint (16) can be reformulated as

\[ EU_V(x_0, x_1) = v(y_V + t - q\ell_0(x_0) - (1 - q)\ell_1(x_1) - R_V) = r, \] (18)
or

\[ y_V + t - q \ell_0(x_0) - (1 - q) \ell_1(x_1) - R_V = v^{-1}(r). \] (19)

The payoff of the injurer can also be formulated in terms of a certainty equivalent. Maximization (14) can be written as

\[
\max_{x_0, x_1} EU_I(x_0, x_1) = u(y_I - nt - qc_0(x_0) - (1 - q)c_1(x_1) - R_I)
\]

subject to eq. (19),

where \( R_I \) is the risk premium due to the uncertainty about the precaution cost. Substituting \( t \) from (19) into (20) yields:

\[
\max_{x_0, x_1} u\left(y_I - qc_0(x_0) - (1 - q)c_1(x_1) - R_I + n\left(y_V - q\ell_0 - (1 - q)\ell_1 - R_V - v^{-1}(r)\right)\right),
\]

which is equivalent to

\[
\max_{x_0, x_1} y_I - qc_0(x_0) - (1 - q)c_1(x_1) - R_I + n\left(y_V - q\ell_0 - (1 - q)\ell_1 - R_V - v^{-1}(r)\right),
\]

since \( u \) is a positive monotone transformation (the slope of \( u(f(x)) \) has the same sign as the slope of \( f(x) \)).

In turn, the latter equation is equivalent to the minimization of ex-ante loss (eq. 4) of Section 3):

\[
\min_{x_0, x_1} \text{L}^{\text{ex-ante}}_{x_0, x_1} = qc_0(x_0) + (1 - q)c_1(x_1) + R_I + n(q\ell_0 + (1 - q)\ell_1 + R_V).
\]

Below, I will prove the statement in Section 2 that a mean preserving contraction in the probabilities of harm at the conventional levels is welfare improving. At the conventional levels, one dollar spent in precaution reduces the cost of harm by one dollar (see eqs. 1 and

\[34\]In the Annex, I pursue the traditional approach and obtain the inequalities driving Proposition 1 in terms of marginal rates of substitutions.
2). If we rewrite the latter equations in terms of probabilities, we get

\[ n_I \frac{\Delta c_0 (x_0^*)}{\Delta x_0} \frac{\Delta x_0^c}{\Delta p_0} = -n_V \left[ h + \frac{\Delta RP^{V}_0 (x_0^*)}{\Delta p_0} \right], \]
\[ n_I \frac{\Delta c_1 (x_1^*)}{\Delta x_1} \frac{\Delta x_1^c}{\Delta p_1} = -n_V \left[ h + \frac{\Delta RP^{V}_1 (x_1^*)}{\Delta p_1} \right], \]

the marginal cost and the marginal benefit of a small increase in the probability of harm should be the same. Thus

\[ q n_I \frac{\Delta c_0 (x_0^*)}{\Delta x_0} \frac{\Delta x_0^c}{\Delta p_0} + (1 - q) n_I \frac{\Delta c_1 (x_1^*)}{\Delta x_1} \frac{\Delta x_1^c}{\Delta p_1} = -q n_V \left[ h + \frac{\Delta R^{V}_0 (x_0^*)}{\Delta p_0} \right] \Delta p_0 - (1 - q) n_V \left[ h + \frac{\Delta R^{V}_1 (x_1^*)}{\Delta p_1} \right] \Delta p_1 \]
\[ -n_V h \left[ q \Delta p_0 + (1 - q) \Delta p_1 \right] - n_V \left[ \frac{\Delta R^{V}_0 (x_0^*)}{\Delta p_0} \Delta p_0 + (1 - q) \frac{\Delta R^{V}_1 (x_1^*)}{\Delta p_1} \Delta p_1 \right]. \]

Since the contraction in the probabilities leaves the mean unchanged, we have \( q \Delta p_0 + (1 - q) \Delta p_1 = 0 \). Furthermore, the total risk borne by the victims should not change. Thus, from (17),

\[ q \frac{\Delta R^{V}_0 (x_0^*)}{\Delta p_0} \Delta p_0 + (1 - q) \frac{\Delta R^{V}_1 (x_1^*)}{\Delta p_1} \Delta p_1 + \left( \frac{\Delta R^{V}_0}{\Delta p_0} \Delta p_0 + \frac{\Delta R^{V}_1}{\Delta p_1} \Delta p_1 \right) = 0. \]

The contraction in the probabilities reduces the wedge between the costs of harm \( \ell_0 (x_0^*) \) and \( \ell_1 (x_1^*) \). Thus \( \left( \frac{\Delta R^{V}_0}{\Delta p_0} \Delta p_0 + \frac{\Delta R^{V}_1}{\Delta p_1} \Delta p_1 \right) < 0 \) and \( \left( \frac{\Delta R^{V}_0}{\Delta p_0} \Delta p_0 + \frac{\Delta R^{V}_1}{\Delta p_1} \Delta p_1 \right) > 0 \). So, we must have, from (21):

\[ q n_I \frac{\Delta c_0 (x_0^*)}{\Delta x_0} \frac{\Delta x_0^c}{\Delta p_0} + (1 - q) n_I \frac{\Delta c_1 (x_1^*)}{\Delta x_1} \frac{\Delta x_1^c}{\Delta p_1} < 0. \]

The contraction reduces the mean precaution expenditure. If injurers are risk neutral, this yields a net welfare gain.

**A2. Optimal damages.** Let us consider again the problem of Section 4. We have

\[ \ell_0^I (x_0) = c_0 (x_0) + p_0 (x_0) n_d_0 + RP^{I}_0 (n_d_0), \]

where \( RP^{I}_0 (d_0) \) is the risk premium attendant with the prospect of bearing liability \( n_d_0 \) with probability \( p_0 (x_0) \). The level of care \( x_0^I \) is decided by the injurer to minimize her cost of harm:

\[ x_0^I = \arg \min_{x_0} (\ell_0^I (x_0)), \]

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and thus
\[ c_0'(x_0^c) = - \left[ p_0'(x_0) nd_0 + \frac{\partial RP_0^V (nd_0)}{\partial x_0} \right]. \]

The impact of an increase in \( nd_0 \) on \( x_0^I \) is generally ambiguous (it is positive, for instance, if risk-aversion is not decreasing in income - see Sweeney and Beard (1992)).

The costs of harm for each victim is:
\[ \ell_0^V (x_0^I) = p_0 (x_0^I) (h - d_0) + RP_0^V (h - d_0), \]

where \( RP_0^V (h - d_0) \) is the risk premium attendant with the prospect of bearing uncompensated harm \( h - d_0 \) with probability \( p_0 (x_0^I) \).

One can easily see that the cost of harm increases if \( h - d_0 \) increases and if \( p_0 (x_0^I) \) decreases. Furthermore, \( RP_0^V (h - d_0) = 0 \) and \( \frac{\partial RP_0^V (h - d_0)}{\partial (h - d_0)} = 0 \) if \( d_0 = h \).

The optimal conventional policy (after ex-ante uncertainty has unfolded) can be found by minimizing social loss (per injurer):
\[ L_0^{SL} (d_0) = \ell_0^I (x_0^I) + n\ell_0^V (x_0^I). \]

We have
\[
\frac{L_0^{SL} (d_0)}{\partial d_0} = \left. \frac{\partial \ell_0^I (x_0^I) \partial x_0^I}{\partial x_0^I \partial d_0} + \frac{\partial \ell_0^I (x_0^I)}{\partial d_0} \right|_{x=x_0^I} + n \left. \frac{\partial \ell_0^V (x_0^I) \partial x_0^I}{\partial x_0^I \partial d_0} + n \frac{\partial \ell_0^V (x_0^I)}{\partial d_0} \right|_{x=x_0^I}
\]

with \( \frac{\partial \ell_0^I (x_0^I)}{\partial x_0^I} = 0 \) because of the optimality of \( x_0^I \).

\[ \text{(1 - p)} v (y - (h - d_0) - RP_0^V) - (1 - p) v (y - (h - d_0)) = 0. \] By implicit differentiation, \( \frac{\partial RP_0^V}{\partial (h - d_0)} = \frac{\partial v'}{(y - p(h - d_0) - RP_0^V)} > 0. \] For \( d_0 = h \), we get \( RP_0^V = 0 \) and \( \frac{\partial RP_0^V}{\partial (h - d_0)} = 0. \)
Note that

\[
\frac{\partial \ell_0^I (x_0^I)}{\partial d_0} \bigg|_{x=x_0^I} = p_0 (x_0^I) n + \frac{\partial R P_0^I (nd_0)}{\partial d_0}
\]

\[
\frac{\partial \ell_0^V (x_0^I)}{\partial d_0} \bigg|_{x=x_0^I} = -p_0 (x_0^I) + \frac{\partial R P_0^V (nd_0)}{\partial d_0}
\]

and

\[
\frac{\partial \ell_0^V (x_0^I)}{\partial x_0^I} \frac{\partial x_0^I}{\partial d_0} = \frac{\partial x_0^I}{\partial d_0} \frac{\partial p_0 (x_0^I)}{\partial d_0} \left[ h - d_0 + \frac{\partial R P_0^V (h - d_0)}{\partial p_0 (x_0^I)} \right].
\]

Thus, upon simplification,

\[
\frac{L_{0}^{SL} (d_0)}{\partial d_0} = \frac{\partial R P_0^I (nd_0)}{\partial d_0} + n \frac{\partial R P_0^V (nd_0)}{\partial d_0} + n \frac{\partial x_0^I}{\partial d_0} \frac{\partial p_0 (x_0^I)}{\partial d_0} \left[ h - d_0 + \frac{\partial R P_0^V (h - d_0)}{\partial p_0 (x_0^I)} \right].
\]

For \(d_0 = h\), the last two terms boil down to nil (see footnote 35). An increase in damages does not affect the payoff of the victims because: i) they can perfectly bear small risks (for very small losses, risk-averse agents behave as risk neutral ones), ii) they do not suffer from an increase in the probability of harm because they are perfectly compensated.

Thus,

\[
\frac{L_{0}^{SL} (d_0)}{\partial d_0} = \frac{\partial R P_0^I (nd_0)}{\partial d_0} > 0.
\]

This proves that optimal conventional damages are under-compensatory (\(d_0^c < h\)).

Let us now see if the same argument applies in the presence of ex-ante uncertainty. From (8), we get

\[
\frac{\partial L^{ex-ante} (d_0, d_1)}{\partial d_0} = q \frac{L_{0}^{SL} (d_0)}{\partial d_0} + \alpha_I q (1 - q) \left[ \ell_0^I (x_0^I) - \ell_1^I (x_1^I) \right] \frac{\partial \ell_0^I (x_0^I)}{\partial d_0}
\]

\[
+ n \alpha_V q (1 - q) \left[ \ell_0^V (x_0^I) - \ell_1^V (x_1^I) \right] \frac{\partial \ell_0^V (x_0^I)}{\partial d_0}.
\]
For \( d_0 = h \) (and thus \( \ell^V_0 (x_0^I) = \ell^V_1 (x_1^I) = 0 \)) we get

\[
\frac{\partial L^{ex-ante} (h, d_1)}{\partial d_0} = q \frac{\partial R \Pi_0 (nh)}{\partial d_0} + \alpha_I q (1 - q) [\ell^I_0 (x_0^I) - \ell^I_1 (x_1^I)] \left[ p_0 (x_0^I) n + \frac{\partial R \Pi_0 (nh)}{\partial d_0} \right].
\]

Damages affect, at the margin, only the risk premium of the injurer. So

\[
\frac{\partial L^{ex-ante} (h, d_1)}{\partial d_0} > 0 \iff \frac{\partial R \Pi_0 (nh)}{\partial d_0} > \alpha_I (1 - q) [\ell^I_1 (x_1^I) - \ell^I_0 (x_0^I)] \left[ p_0 (x_0^I) n + \frac{\partial R \Pi_0 (nh)}{\partial d_0} \right],
\]

with \( p_0 (x_0^I) n + \frac{\partial R \Pi_0 (nh)}{\partial d_0} > 0 \).

Inequality (22) might not be met. In particular, if state 0 represents a positive shock for the injurer, \( \ell^I_1 (x_1^I) > \ell^I_0 (x_0^I) \), that takes place with small probability (small \( q \)), then over-compensatory damages are likely to be optimal.

A3. Ambiguity aversion (regulatory standards). There are several models of ambiguity aversion. Here, I use one of the most general, the smooth model of Klibanoff et al. (2005), which posits that parties are averse to mean preserving spreads of their beliefs.

The victims’ ex-ante welfare can be written as

\[
W = q \varphi \left( \widehat{EU}_0^V (x_0) \right) + (1 - q) \varphi \left( \widehat{EU}_1^V (x_1) \right),
\]

where \( \varphi \) is a concave monotone function. As with standard risk aversion, the concave transformation implies that the welfare derived from the two expected utilities is less than the mean value of the expected utilities.

From (23), the uncertainty premium due to ambiguity, \( R^A \), should meet

\[
W = \varphi \left( q \widehat{EU}_0^V (x_0) + (1 - q) \widehat{EU}_1^V (x_1) - R^A \right)
= \varphi \left( q v (y_V - \ell_0) + (1 - q) v (y_V - \ell_1) - R^A \right).
\]

So, the level of welfare achieved by the victims can be seen as a monotone transformation of conventional expected utility minus an ambiguity premium. The ambiguity premium \( R_A \) decreases if the wedge between \( \ell_0 (x_0) \) and \( \ell_1 (x_1) \) decreases.
In view of (18), welfare can be also written as
\[ W = \varphi \left( y_V - q \ell_0 - (1 - q) \ell_1 - R_V - R^A \right). \]

So, the maximization of \( W \) is equivalent to the minimization of ex-ante loss:
\[ \frac{\bar{\ell}^{\text{ex-ante}}}{V} (x_0, x_1) = q \ell_0 (x_0) + (1 - q) \ell_1 (x_1) + R_V + R^A. \]

The optimal ex-ante policy should thus minimize
\[ \frac{\bar{L}^{\text{ex-ante}}}{I} (x_0, x_1) = n_I \ell_I^{\text{ex-ante}} (x_0, x_1) + n_V \frac{\bar{\ell}^{\text{ex-ante}}}{V} (x_0, x_1) \]
\[ = q L_0 (x_0) + (1 - q) L_1 (x_1) + n_I R_I + n_V (R_V + R^A). \]

Thus
\[ \frac{\partial L^{\text{ex-ante}}}{\partial x_0} (x_0, x_1) = q L_0' (x_0) + n_I \frac{\partial R_I}{\partial c_0} (x_0) c_0' (x_0) + n_V \frac{\partial (R_V + R^A)}{\partial \ell_0} \ell_0' (x_0). \]

At \( x_0 = x_0^c, x_1 = x_1^c, \) (using \( L_0' (x_0) = 0 \) and \( n_I c_0' (x_0^c) = -n_V \ell_0' (x_0^c) \)), we get
\[ \frac{\partial L^{\text{ex-ante}}}{\partial x_0} (x_0^c, x_1^c) > 0 \Leftrightarrow \frac{\partial R_I}{\partial c_0} (x_0^c) > \frac{\partial R_V}{\partial \ell_0} (x_0^c) + \frac{\partial R^A}{\partial \ell_0} (x_0^c). \] (24)

Both \( R_V \) and \( R^A \) increase as the wedge between \( \ell_0 (x_0) \) and \( \ell_1 (x_1) \) increases. So, the addendum \( \frac{\partial R^A}{\partial \ell_0} (x_0^c) \) just amplifies the cost of ex-ante uncertainty. It makes attenuation more desirable.

**A4. Liability vs. regulation.** Let us assume that victims are risk (and ambiguity) neutral. Under liability for harm, ex-ante social loss is (omitting arguments)

\[ L^{\text{ex-ante}} (d_0^*, d_1^*) = \ell_I^{\text{ex-ante}} (d_0^*, d_1^*) + n \frac{\ell_V^{\text{ex-ante}}}{V} (d_0^*, d_1^*) = \]
\[ q (c_0 + p_0 n d_0 + R P_0) + (1 - q) (c_1 + p_1 n d_1 + R P_1) + R_I \]
\[ + n \left[ q (p_0 (h - d_0)) + (1 - q) (p_1 (h - d_0)) \right] \]
where \((d^*_0, d^*_1)\) are the optimal damages.

Let us suppose that the level of precaution \(x^L_0, x^L_1\) arising under liability are used as standards in the regulation regime (where the loss fully lies on the victims). Ex-ante social loss becomes

\[
L^{\text{ex-ante}}(x^L_0, x^L_1) = q c_0 + (1 - q) c_1 + R_{I}^{\text{reg}} + n [q p_0 h + (1 - q) p_1 h].
\]

So, \(L^{\text{ex-ante}}(d^*_0, d^*_1) > L^{\text{ex-ante}}(x^L_0, x^L_1)\) if

\[
q R P_0 + (1 - q) R P_1 + R_I > R^{\text{reg}}_I.
\]

So, a sufficient condition for regulation to be socially preferable is \(R_I > R^{\text{reg}}_I\).

An Arrow-Pratt approximation yields:

\[
R_I > R^{\text{reg}}_I \iff \frac{1}{2} q (1 - q) \alpha_I [c_0 + p_0 d_0 + R P_0 - c_1 - p_1 d_1 - R P_1]^2 > \frac{1}{2} q (1 - q) \alpha_I [c_0 - c_1]^2 \iff [c_0 - c_1 + p_0 d_0 + R P_0 - p_1 d_1 - R P_1]^2 > [c_0 - c_1]^2
\]

(25)

Let \(c_0 > c_1\). Then inequality (25) is met if \(p_0 d_0 + R P_0 \geq p_1 d_1 + R P_1\). Let \(c_0 < c_1\). Then inequality (25) is met if \(p_0 d_0 + R P_0 \leq p_1 d_1 + R P_1\).

So, a sufficient condition for the superiority of regulation is that the payment of damages in addition to precaution costs increases ex ante uncertainty.
References


Annex

Optimal standards. In what follows, I take a different route from that of Section 3 and work directly with utility functions (instead of risk premiums). Let us consider a change in $x_0$. From (16), we get that the implicit level of the transfer, $t = t(x_0, x_1)$, must satisfy:

$$\frac{\partial t_0 (x_0, x_1)}{\partial x_0} = -\frac{q \frac{\partial \bar{EU}_0^V (x_0)}{\partial t_0}}{q \frac{\partial \bar{EU}_0^V (x_0)}{\partial t_0} + (1 - q) \frac{\partial \bar{EU}_1^V (x_1)}{\partial t}} < 0,$$

(if the standard $x_0$ increases, the transfer decreases).

By using (26), we get from (14):

$$\frac{\partial EU_I(x_0, x_1)}{\partial x_0} = \frac{\partial EU_I(x_0, x_1)}{\partial t} \frac{\partial t_0 (x_0, x_1)}{\partial x_0} + \frac{dEU_I(x_0, x_1)}{dx_0} \frac{q \frac{\partial \bar{EU}_0^V (x_0)}{\partial x_0}}{q \frac{\partial \bar{EU}_0^V (x_0)}{\partial t} + (1 - q) \frac{\partial \bar{EU}_1^V (x_1)}{\partial t}} - q u^I(y_I - nt - x_0).$$

So, $\frac{\partial EU_I(x_0, x_1)}{\partial x_0} > 0$ if, and only if:

$$\frac{q u^I(y_I - nt - c_0(x_0)) + (1 - q) u^I(y_I - nt - c_1(x_1))}{q u^I(y_I - nt - x_0)} n > \frac{q \frac{\partial \bar{EU}_0^V (x_0)}{\partial t} + (1 - q) \frac{\partial \bar{EU}_1^V (x_1)}{\partial t}}{\frac{q \frac{\partial \bar{EU}_0^V (x_0)}{\partial x_0}}{q \frac{\partial \bar{EU}_0^V (x_0)}{\partial t} + (1 - q) \frac{\partial \bar{EU}_1^V (x_1)}{\partial t}}}.$$

When $q = 1$ (the conventional case), we get that $\frac{\partial EU_I(x_0, x_1)}{\partial x_0} = 0$ if $x_0^c$ satisfies (from 27):

$$1 = \frac{n \frac{\partial \bar{EU}_0^V (x_0)}{\partial t}}{\frac{\partial \bar{EU}_0^V (x_0)}{\partial x_0}},$$

i.e., $n \frac{\partial \bar{EU}_0^V (x_0)}{\partial x_0} = \frac{\partial \bar{EU}_0^V (x_0)}{\partial t}$.  

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So, if \( x_0 = x_0^c \) and \( x_1 = x_1^c \), we get that 
\[
\frac{\partial EU_1(x_0^c, x_1^c)}{\partial x_0} > 0 \iff 
1 + \frac{(1 - q) u'(y_I - nt - c_1(x_1^c))}{q u'(y_I - nt - c_0(x_0^c))} > \frac{\partial EU_0^V(x_0^c)}{\partial t} + (1 - q) \frac{\partial EU_1^V(x_1^c)}{\partial t} \]
\[
1 + \frac{(1 - q) u'(y_I - nt - c_1(x_1^c))}{q u'(y_I - nt - c_0(x_0^c))} > 1 + \frac{(1 - q) \partial EU_1^V(x_1^c)}{q \partial EU_0^V(x_0^c)} \iff 
\frac{u'(y_I - nt - c_1(x_1^c))}{u'(y_I - nt - c_0(x_0^c))} > \frac{\partial EU_1^V(x_1^c)}{\partial t} \frac{\partial EU_0^V(x_0^c)}{\partial t}.
\] (28)

Following the same steps, one gets
\[
\frac{\partial EU_1(x_0^c, x_1^c)}{\partial x_1} > 0 \iff \frac{u'(y_I - nt - c_1(x_1^c))}{u'(y_I - nt - c_0(x_0^c))} < \frac{\partial EU_1^V(x_1^c)}{\partial t} \frac{\partial EU_0^V(x_0^c)}{\partial t}.
\] (29)

Ineq. (28) and (29) are the equivalent of the inequalities in Proposition 1. The marginal utility ratios of ineq. (28) provide a measure of the risk exposure of the parties. If injurer and victims were risk neutral, or if they could purchase insurance at a fair price, both ratio of ineq. (28) would be equal to 1. Because of the concavity of the utility functions and the lack of insurance, both the RHS and the LHS of (28) are less than unity. The more concave the utility function of a party, and the smaller her marginal utilities ratio (given \( x_0^c \) and \( x_1^c \)).

Note that \( x_0^c \) and \( x_1^c \) do not depend on the degree of risk aversion of the injurer. So, given the functions \( v \) and \( u \), one can always find a (sufficiently) concave transformation of \( u \) such that inequality (28) is met. Conversely, if the transformation is convex, (29) is met.